On the Kutta Condition for Sound Transmission in an Annular Cascade of Vanes

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Long Abstract

Introduction

The present work deals with the analytical formulation of sound transmission through the outlet guide vanes of a subsonic axial-flow fan. Drastic approximations on the geometry and the flow features are necessary in order to produce tractable, closed-form solutions. Because the exit flow from outlet guide vanes must be axial, the vanes are nearly parallel to the axis at the trailing-edge. They can be modeled as an annular cascade of axially aligned and zero-thickness plates. This simplification remains representative of a system of non-parallel vanes with hub and carter rigid-wall boundary conditions. The flow is considered as homoentropic and inviscid of uniform axial Mach number M. Previous studies by Howe [1] and Rienstra [2] highlighted the importance of the Kutta condition in the formulation of such sound transmission problems. Recently, Bouley *et al.* [3] showed that, in a 2D reduction of the same problem, the scattered acoustic fields can be significantly overestimated when this condition is not imposed.

The present work contributes by including a Kutta condition in an original formulation based on the mode-matching technique detailed by Ingenito *et al.* [4].

1. Methods

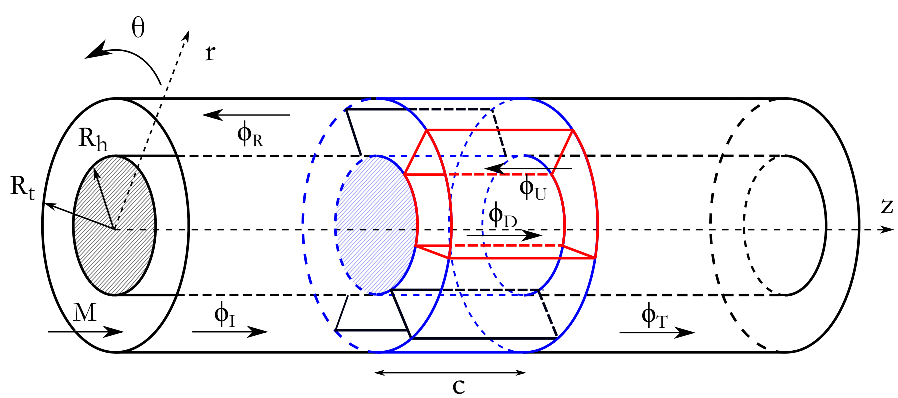

Figure 1. Annular cascade of vanes, modeled by zero-thickness flat plates, impinged by an incident acoustic wave of potential ϕ_I. Scattered potentials for each subdomain are denoted ϕ_R,ϕ_D,ϕ_U and ϕ_T.

1.1 Mode-matching technique

The mode-matching technique, detailed by Mittra & Lee [5] for electromagnetic boundary-value problems, relies on two steps:

1. The scattered potentials are known in various subdomains (annular duct upstream and downstream the stator, inter-vane channels) as sum of modes, each of which satisfies the convected Helmholtz equation and the rigid-wall conditions of its subdomain. The modal amplitudes are the unknowns of the problem.

2. Matching equations on the acoustic pressure and axial velocity are written at the leading-edge interface ($z = 0$) and at the trailing-edge interface ($z = c$) as

$$\begin{aligned} \mathbf{\Gamma}_I(r,\theta,0^-) + \mathbf{\Gamma}_R(r,\theta,0^-) &= \mathbf{\Gamma}_D(r,\theta,0^+) + \mathbf{\Gamma}_U(r,\theta,0^+), \quad \forall r \in [R_h,\ R_t], \quad \forall\theta, \quad (1) \\ \mathbf{\Gamma}_D(r,\theta,c^-) + \mathbf{\Gamma}_U(r,\theta,c^-) &= \mathbf{\Gamma}_T(r,\theta,c^+), \quad \forall r \in [R_h,\ R_t], \quad \forall\theta, \quad (2) \end{aligned}$$

introducing the pressure-axial velocity vector:

$$\mathbf{\Gamma} = [\ p \quad v_x\ ]^T$$

R_h and R_t denote the hub and tip radii, respectively. Using mathematic projections, these matching equations can be transformed into a square linear system. The modal amplitudes are then found by matrix inversion after truncation.

1.2 Kutta condition in an annular formalism

The Kutta condition is fulfilled by imposing a zero-pressure jump between both sides of each vane at the trailing-edge:

$$\Delta p(r,\theta = 2\pi m/V, z = c^-) = 0, \qquad \forall r \in [R_h,\ R_t], \quad m \in \mathbb{Z} \tag{3}$$

where V is the number of vanes. Solving the aforementioned matching equations with this condition leads to an over-determined linear system.

For physical and mathematical consistencies, vorticity perturbations downstream of the vanes should be added to introduce viscosity effects. Previous works by Howe [6] suggest that the shed vorticity is concentrated on lines in a two-dimensional context downstream the trailing-edges for zero-thickness plates. These perturbations can be modeled for the present application as thin vortex sheets, located at $\theta = 2\pi m/V$ and convected by the mean flow. Assuming that the associated vorticity is dominant in the radial direction, it is expressed as

$$\mathbf{\Omega}^K(r,\theta,x).\mathbf{r} = \sum_{\mu\in\mathbb{N}} \Omega_\mu f_{n\mu}(r) \times \sum_{m\in\mathbb{Z}} \mathrm{e}^{imu}\delta\left(\theta - \frac{2\pi m}{V}\right) \times \mathrm{e}^{\mathrm{i}(k/M)z}, \quad u = \frac{2\pi n}{V}, \quad k = \frac{\omega}{c_0} \tag{4}$$

The radial vorticity is expressed as the product of three functions in terms of separable variables. The function of z explicits the convection by the mean flow. The azimuthal function is built with a Dirac comb, each Dirac function being phase-shifted between adjacent trailing-edges, n being the azimuthal order of the incident wave. Each radial fonction $f_{n\mu}$ is a combination of Bessel functions, weighted by the unknowns Ω_μ. These unknowns are determined by inserting the axial velocity associated with the vortex sheet into the matching equations at the trailing-edge interface (Eq. 2). The simultaneous solving of Eq. 2 to 4 leads to a square linear system that is now well-posed and can be reduced by matrix inversion.

2. Conclusions

The proposed three-dimensional mode-matching approach has already been applied to formulate the scattering problem on the cascade of vanes. The implementation of the Kutta condition is presently in progress. The work is an extension of the two-dimensional procedure described by Bouley *et al.* [3].

References

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