Combined Analytical Models for Sound Generation and Transmission in Cambered Axial-Flow Outlet Guide Vanes

Michel Roger, Laboratoire de Mécanique des Fluides et Acoustique, Ecole Centrale de Lyon, Ecully, France

Long Abstract

Introduction

Sound generation and transmission mechanisms in blade/vane rows of axial-flow turbomachines need being understood and efficiently predicted for the sake of low-noise strategies. In this context analytical techniques are very attractive at the early design stage. The present analytical work addresses the acoustic response of cambered outlet guide vanes to either acoustic or hydrodynamic waves (wakes) generated by an upstream rotor. The architecture is fully subsonic and depicted in Fig.1-left. The problem is formulated in the frequency domain. It involves two steps repeated in an iterative procedure, in such a way to include the effect of vane camber in the analysis. The first step is a mode-matching technique that determines the response of either the (staggered) leading-edge or the (not staggered) trailing-edge interface. The second step reproduces sound propagation in the slowly-varying cross-section of the inter-vane channels. The implementation is described in a two-dimensional unwrapped representation of the stator annulus in order to demonstrate the feasibility but the method can be extended in cylindrical coordinates to model an annular cascade.

1. Methods

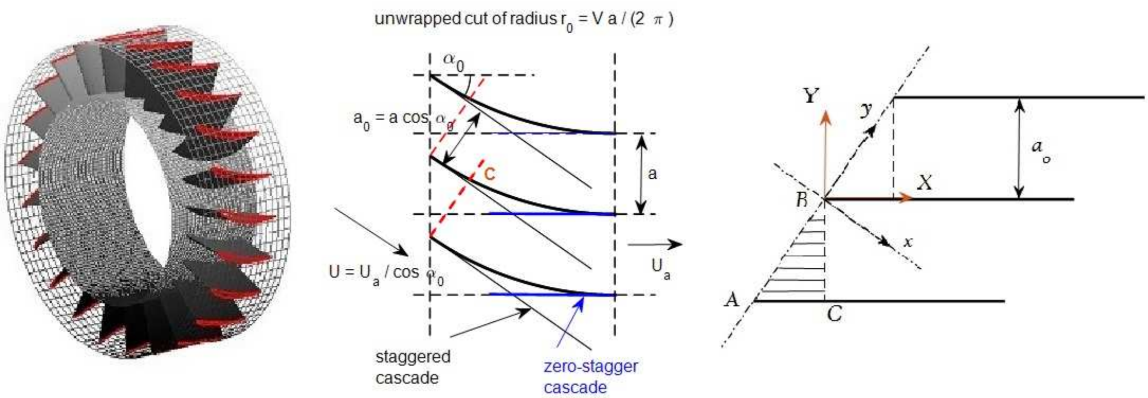

Figure 1. Typical axial-flow outlet guide vanes (left), unwrapped two-dimensional cascade representation (middle) and reference frames for Whitehead's formulation (right, opposite stagger angle).

Acoustic Scattering - The diffraction of an incident acoustic wave at the leading-edge front of the stator is first modeled with Whitehead's approach [1] without any wave coming from downstream in the inter-vane channels. The incident, reflected and transmitted waves are described by their velocity potentials. Two reference frames are introduced, one of coordinates (x, y) along the leading-edge interface and another one (X, Y) with origin at the edge of a plate taken as reference and the X axis along the plate (Fig.1-right). The incident wave is written as

$$\phi_i = A\,\mathrm{e}^{\mathrm{i}\,k\,(\sin\theta_i\, y+\cos\theta_i\, x)} = A\,\mathrm{e}^{\mathrm{i}\,(k_y y+k_x x)}$$

with

$$k_y = k\sin\theta_i = \frac{j\,2\pi}{Va} = \frac{j\,2\pi}{V a_0}\cos\alpha_0 \qquad k_x^2 = k^2 - \left(\frac{j\,2\pi}{Va_0}\right)^2\cos^2\alpha_0 = k_j^2$$

V being the number of vanes and k the acoustic wavenumber. Here $a = a_0/\cos\alpha_0 = 2\pi\, r_0$ if r_0 is the radius of the annular cut. The reflected wave reads

$$\phi_r = \sum_{s=-\infty}^{\infty} A_s\,\mathrm{e}^{\mathrm{i}\,[2\pi/(Va)]\cos\alpha_0\,(j+sV)y}\,\mathrm{e}^{-\mathrm{i}\,k_s x} \qquad k_s^2 = k^2 - \left(\frac{(j+sV)2\pi}{Va_0}\right)^2\cos^2\alpha_0\,.$$

with $-\mathrm{i}\,k_s = \gamma_s$ (or $k_s = \mathrm{i}\,\gamma_s$). The transmitted wave in the reference channel (the phase shift between adjacent channels is $j\,2\pi/V$) is expressed in the coordinates (X,Y) as

$$\phi_t = \sum_{m=0}^{\infty} B_m \cos\frac{m\pi\,Y}{a_0}\,\mathrm{e}^{\mathrm{i}\,K_m X} \qquad K_m^2 = k^2 - \left(\frac{m\,\pi}{a_0}\right)^2; \quad K_m = i\,\Gamma_m\,.$$

The connection between both sides of the matching triangle is ensured by Green's reciprocal theorem with two sets of functions G ensuring zero contribution of the side BC to the integral:

$$\oint_T \left(G\frac{\partial\phi}{\partial n} - \phi\frac{\partial G}{\partial n}\right) d\eta = 0 \qquad G_n^{(1)} = \cos\frac{n\pi\,Y}{a_0}\,\mathrm{e}^{-\Gamma_n X}\,; \quad G_n^{(2)} = \cos\frac{n\pi\,Y}{a_0}\,\mathrm{e}^{+\Gamma_n X}\,.$$

where T denotes the triangular contour, $\partial/\partial n$ the normal derivative and G any function that satisfies the rigid-wall condition.

The theorem is applied with $\phi = \phi_i + \phi_r$ along AB and with $\phi = \phi_t$ along AC, generating matching equations. The latter are equivalent to the continuity of fluctuating pressure and normal velocity usually referred to in acoustics. Once solved the matching equations provide in particular the coefficients of excited waves inside the channels. These waves propagate experiencing the continuous variation of the channel height, which is accounted for in the modeling using Rienstra's formulation [2]. At this step the true location of the point C in Fig.1-middle is shifted onto the tangent to the mean-camber line at leading edge; the approximation makes sense for many stator architectures. Then another matching problem is solved at the trailing edge in order to determine the back-reflected waves and the waves transmitted downstream. The former lead to a new formulation of the leading-edge matching equations that are solved again, and so on. Back-and-forth iterations are computed till convergence, leading to a uniformly valid description of the sound field.

Hydrodynamic Scattering - The acoustic response of outlet guide vanes to incident hydrodynamic disturbances can be determined in the same way at the price of introducing an additional equation for the transport of vorticity (see Bouley *et al* [3] for the zero-stagger and zero-camber stator). An alternative and simpler approach is proposed here. An edge dipole is first defined according to which the impingement of vortical disturbances on a leading-edge is replaced by an equivalent acoustic excitation. The relevance of this equivalence is confirmed by Roger & Moreau [4] for the similar case of stator trailing-edge noise. It is based on the use of the exact half-plane Green's function and avoids resorting to the vortical field explicitly. In a second step the direct field of the edge dipole is expanded into a set of acoustic plane waves that are taken as incident conditions on the stator front face. The scattering is calculated by the aforementioned method.

The full-length paper will detail the derivations and show sample application results. The stagger angle at leading edge is a key factor in the definition of upstream-propagating waves and in the assessment of the resonance properties of the stator. Moreover the present model accounts for the mean-flow deflection between upstream and downstream, often ignored in most analytical studies.

References

[1] E.A.N. Whitehead. The Theory of Parallel-Plate Media for Microwave Lenses, *Proc. IEE London*, **98** (part III), pp. 133-140, 1951.

[2] S. Rienstra. Sound transmission in slowly varying circular and annular lined ducts with flow, *J. Fluid Mech*. **380**, pp. 279-296, 1999.

[3] S. Bouley, B. François, M. Roger & S. Moreau. On a mode-matching technique for sound generation and transmission in a linear cascade of outlet guide vanes, *Aviation Forum 2015*, Dallas, 2015.

[4] M. Roger & S. Moreau. Towards Cascade Trailing-Edge Noise Modeling Using a Mode-Matching Technique, *Aviation Forum 2015*, Dallas, 2015.