Numerical Analyses for Cavitation Surge in a Pump with the Square Root Shaped Suction Performance Curve

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Long Abstract

Introduction

The suction performance curves of the hydraulic pumps show several characteristics such as a sudden drop and gradual decay of pump head and so on. In some particular cases, suction performance curves have a square root shape of "√" that shows a drop followed by a short term rise and another drop of pump head . In the present study cavitation surge under the square root shaped curve is analyzed by using numerical analsyses. Two methods of a lumped parameter system calculation and a distributed parameter system calculation that is in other words three dimensional computation of Navier Stokes Equations are applied. In the case of a lumped parameter system calculation, pump static pressure rise is assumed on the given square root shaped function. Dynamic behaviors of the pump cavitation are represented by the cavitation compliance and the mass flow gain factor. Ordinary Differential Equations are discretized by the second order Runge-Kutta method.In the case of 3D CFD, a commercial code of ANSYS-CFX with cavitaion model is adopted.

Methods and Results

Objective pump sysyem is shown in Fig.1 The square root shaped suction performance curve of this pump is shown in Fig.2. In the case of a lumped parameter system calculation, geoverning equations are as follow.

$$P_{T1} = \text{const.} \qquad (1)$$

$$P_{T2} = \text{const.} \qquad (2)$$

$$\rho L_1 \frac{dQ_1}{dt} = S_1(P_{T1} - P_S) - \frac{\lambda_1 \rho L_1 Q_1^2}{2D_1 S_1} \qquad (3)$$

$$\rho L_2 \frac{dQ_2}{dt} = S_2(P_P - P_{T2}) - \frac{\lambda_2 \rho L_2 Q_2^2}{2D_2 S_2} \qquad (4)$$

where P is static pressure, Q is volume flow rate, L is length of a pipe, S is cross sectional area of a pipe, D is diameter of a pipe, ρ is density of water and λ is Darcy's skin friction coefficient respectively. Subscripts of *1* is a pump upsteram pipe, *2* is a pump downstream pipe, *S* is inlet of a pump, *P* is outlet of a pump, *T1* is a tank number one and *T2* is a tank number two.

$$P_P - P_S = F(Q, P_S) \approx D_{PQ}(Q - Q_0) + F(Q_0, P_S) \qquad (5) \qquad where \quad D_{PQ} = \left.\frac{\partial F}{\partial Q}\right|_{Q=Q_0}$$

F is the prescribed function of the the square root shaped suction performance curve of static pressure rise as seen in Fig.2 Q_0 is the voulme flow rate at the initial operating point.

$$\frac{dV_C}{dt} = \frac{\partial V_C}{\partial P_S}\frac{\partial P_S}{\partial t} + \frac{\partial V_C}{\partial Q_1}\frac{\partial Q_1}{\partial t} = \frac{\partial V_C}{\partial P_S}\frac{dP_S}{dt} + \frac{\partial V_C}{\partial Q_1}\frac{dQ_1}{dt} = -K\frac{dP_S}{dt} - M\frac{dQ_1}{dt} \qquad (6)$$

$$K = -\frac{\partial V_C}{\partial P_S} \qquad (7)$$

$$M = -\frac{\partial V_C}{\partial Q_1} \qquad (8)$$

V_c is cavitation volume and K and M are cavitation compliance and mass flow gain factor respectively.

$$Q_1 - Q_2 = -\frac{dV_C}{dt} \qquad (9)$$

Following Eq.10 is developed from Eq.6 and Eq.9

$$\frac{dP_S}{dt} = \frac{1}{K}\left(Q_1 - Q_2 - M\frac{dQ_1}{dt}\right) \qquad (10)$$

Equations 3, 4 and 10 are computed in time domain by using the second order Runge Kutta method. Computed results are seen in Figures 3. Computed results by using ANSYS-CFX are also presented in the full paper.

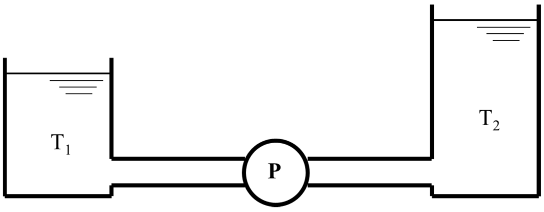


Fig.1 Objective Model of a Pump System

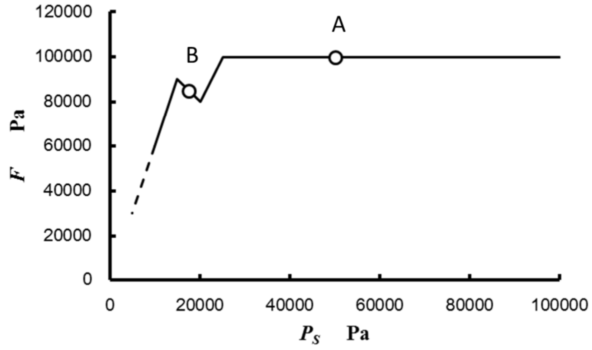
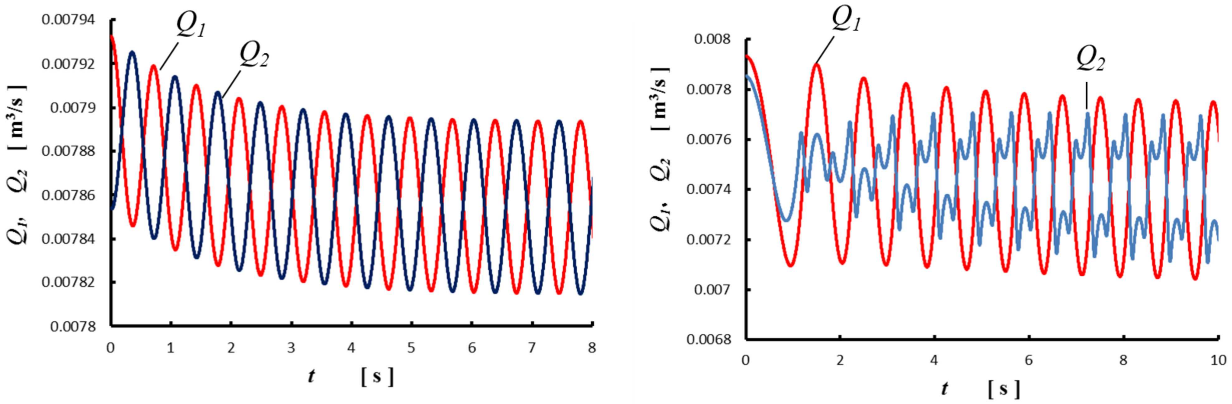


Fig.2 A Square Root Shaped Suction Performance Curve (Q=Q_0)



(a) The Flat Suction Performance Curve Case of Point A (M>0)

(b) The Square Root Shaped Suction Performance Curve Case of Point B (M=0)

Fig.3 Computed Results of the Time History of Volume Flow Rate under Cavitation Surge