DYNAMIC ANALYSIS OF A BEAM WITH PERIODICALLY VARYING BENDING STIFFNESS WITH AN AXIALLY MOVING MASS

Long Abstract

Helmut J. Holl, Institute of Technical Mechanics, University of Linz, Linz, Austria

Introduction

The transversal vibrations of an elastic Euler- Bernoulli beam with periodically varying bending stiffness loaded by an axially moving single mass is presented. The single point mass travels along the beam with a given speed, however it is connected to the beam by a spring, which causes vibrations of the mass relative to the beam. For this case the equations of motion are derived, the eigenfrequencies of the system are investigated and solutions for the transverse displacement, shear forces and bending moments are presented. Various periodicity of the beam stiffness is investigated in some detail. A numerical study is presented for a representative beam, to show the sensitivity to characteristic system parameters.

1. Mechanical Model

Starting from the equations of motion of an Euler- Bernoulli beam a variable stiffness is introduced and additionally a point mass is attached by a spring to the beam. It is assumed that the point mass moves axially with a constant speed. Additionally an excitation force can be considered which acts on the concentrated point mass in transversal direction of the beam. For the axially variable stiffness of the beam a series expansion is formulated and inserted in the differential equations. Further the problem was described in a non-dimensional form in order to cover all possible cases by a single formulation. A Ritz-Galerkin procedure is used in order to solve the differential equation of motion. Convergence of the solution is improved by separation of a static and a dynamic part of the solution. The section forces and moments for the system with the moving concentrated mass are computed using the derivatives of the solution of the transversal oscillations with respect to the longitudinal coordinate. As an analytic formulation is used, timeintegration is done by solving the convolution integral. The eigenfrequencies of the beam are evaluated for various parameters of the periodically variable stiffness.

2. Computed Results

The symbolic computer codes Maple and MatLab are used to solve the problem under consideration. In order to illustrate the influence of the variable bending stiffness upon the deflection of the beam a series of symbolic computations are performed. For the numerical studies of the dynamic system with an axially transported mass deflection, section forces and moments are given for different periodicities of the variable stiffness of the beam and different axial speeds of the concentrated mass. Additionally special emphasis is given to the ratio of the concentrated mass and the mass of the beam.

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References

[1] Baker, W.E., Westine, P.S., Dodge, F.T. 1991, Similarity Methods in Engineering Dynamics. Theory and Practice of Scale Modeling, Revised Edition, Elsevier Science Publ., Amsterdam, NL.

[2] Fryba, L., 1999, Vibrations of Solids and Structures under Moving Loads, Thomas Telford, London, GB.

[3] Ziegler, F., 1991, Mechanics of Solids and Fluids, Springer-Verlag, New York, USA.