Nonsmooth modes from elementary spring-mass systems to turbomachines: current limitations

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Long Abstract

Introduction

Nonlinear modes provide useful means of studying the dynamical behaviour of nonlinear systems [1, 2, 3]. Indeed, they allow for a description of the system in the neighbourhood of a linear mode. Once generalized coordinates are introduced, they can be represented as bidimensional surfaces in the phase space called *invariant manifolds*, which are constituted by a continuum of periodic trajectories. *Manifold* refers to the mathematical object while *invariant* translates the fact that if the generalized coordinates and velocities lie at some time on the manifold, it will remain on it forever. Such nonlinear modes have been applied to turbomachines (??) but only apply to systems which can be described by ODEs.

Blades in turbomachines may undergo contact and impact with the abradable material [5]; such events may no longer be described with ODEs and require more complex mathematical tools such as linear complementary conditions, inclusions in cones, inclusions of measure differential equations or variational inequalities [4]. Such systesm, undergoing nonsmooth events, are called nonsmooth systems. Nonsmoothness refer to velocities which are discontinuous during impacts. Nonlinear modal techniques can therefore not be directly extended to nonsmooth systems. It is noteworthy that even though important, there is no modal prediction tools on such systems.

In [CITE IDETC2015], the extension of nonlinear modes to nonsmooth modes have been studied on a 2-dof spring-mass system. The oscillator was subject to unilateral contact with a Newton impact law. The existence of non-smooth invariant manifolds has been illustrated for two types of periodic motions: those involving a single impact for each period, and those involving two impacts per period. A nonsmooth mode for the latter case is represented in Figure 1, for illustration purposes.

The aim of this paper is to study how such methods of nonsmooth modal analysis can be of interest for turbomachines, for which usual nonlinear modal techniques no longer apply when blade–casing contact occur. More precisely, several questions are open: can they apply to turbomachines in theory? In practice? Can they provide efficient tools for the dynamic behaviour, simarly to those offered by nonlinear modes?

Methods and first results

Turbomachine models include at least dozens of thousands of nodes. For this reason, the method of [IDETC] was first extended from 2 to larger numbers of degrees of freedom, see Figure 2. This is straightforward for motions with one impact per period where solutions exist for any given period T. For two impacts per period, solutions only exist for some times of impact t_1, t_2 so an additional sophisticated step is to find appropriate times. This is done by solving numerically a closed-form expression. Using a standard computer, we managed to identify some for 70 degrees-of-freedom.

Such an extension additionally allows for the convergence study with respect to spatial discretisation. This part is currently in progress and consists in measuring the change of the nonsmooth modes for a small change of the number of degrees-of-freedom.

FEM models used for turbomachines involve *a priori* non-diagonal mass matrices, which makes this

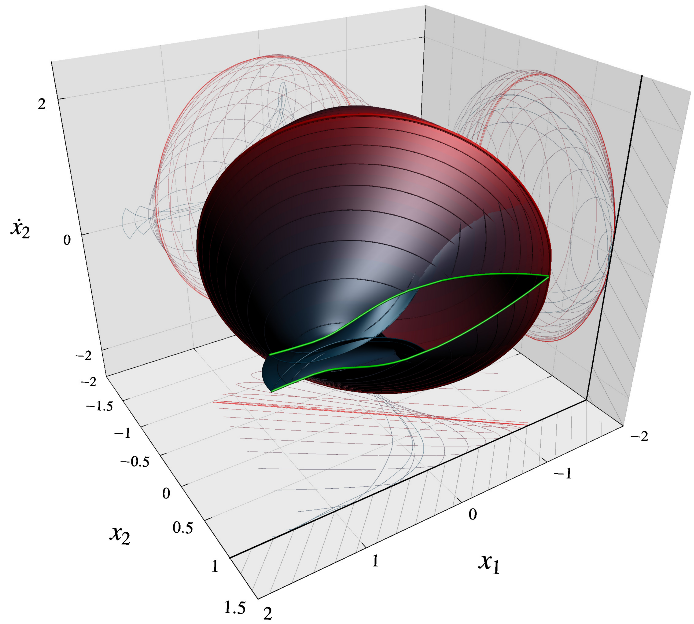

Figure 1. Illustration of a nonsmooth mode: continuum of orbits in the phase space. The red ellipse corresponds to a linear mode.

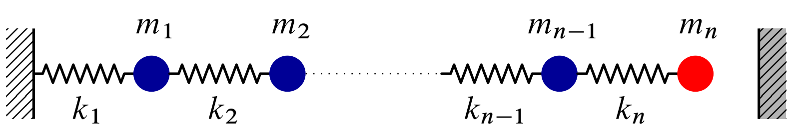

Figure 2. Base model for the study of nonsmooth modes (m_i are masses and k_i are stiffnesses).

method no directly applicable. Nevertheless, the mass matrix can be lumped. This results in a diagonal matrix so the global structure of the dynamic matrix equation is similar to that of the spring-mass system.

The stability of the manifolds is then studied. First, stability to a small change in the initial conditions is investigated (Lyapunov stability and asymptotic stability). Then, orbital stability is considered. Lastly, sensivity to the physical parameters k_i and m_i is examined.

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