Meandering of turbine wake flow induced by atmospheric eddies

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Long Abstract

Introduction

The wake flow of a wind turbine features velocity deficit owing to the energy extraction. As a result a downstream turbine experiences lower-speed wind and generates less power. The oscillation of a wake flow, i.e. wake meandering, shakes the deficit wind away from the downstream turbine and improves the wind farm efficiency. Such a wake meandering can be induced by either intrinsic wake instabilities, e.g. the instability of helical vortices. However, the signature of tip vortices is only significant up to 1-2 rotor diameters downstream of the turbine as revealed in a wind tunnel experiment [1]. Another candidate mechanism of wake meandering is the development of large-scale free-stream eddies. It is assumed that small-scale eddies (i.e. smaller than the rotor diameter), constituting the high-frequency part of the turbulence spectrum, are responsible for diffusive effects in the wake only, whereas the low-frequency part, composed of eddies larger than the rotor diameter, contributes mainly to transport the wake as a whole. In this work, the most energetic inflow perturbations is calculated to investigate the influence of upstream noise to the wake flow development.

In most of the perturbation studies, e.g. stability, non-normality, sensitivity, to name a few, the linearized Navier-Stokes (NS) equation and its adjoint are solved, with a limited number of nonlinear exceptions, mainly in pipe flow. In this work, a three-dimensional nonlinear solver to calculate the optimal inflow noise is developed based on a well established linear counterpart [2]. The numerical challenge of such a nonlinear calculation is that the developing history of the flow, which easily exceeds one terabytes, has to be saved for the integration of an adjoint equation. The benefit of this nonlinear calculation is that large perturbations can be taken into account, and modes with various azimuthal wavenumbers can be studies in a single simulation.

Methods

An actuator-disc model is adopted to represent the wind turbine. Assuming that the turbine works at the Betz limit, the wind flow is governed by the NS equation with a body force term:

$$\partial_t \boldsymbol{u} + \boldsymbol{u}\cdot\boldsymbol{\nabla}\boldsymbol{u} + \boldsymbol{\nabla} p - Re^{-1}\nabla^2 \boldsymbol{u} = -\frac{(\boldsymbol{f}\cdot\boldsymbol{u},\boldsymbol{f}\cdot\boldsymbol{u})}{3(\boldsymbol{f},\boldsymbol{f})^2}\boldsymbol{f}, \quad \text{with} \quad \nabla\cdot\boldsymbol{u} = 0, \tag{1}$$

where $(\boldsymbol{a},\boldsymbol{b})$ is defined as the integration of $\boldsymbol{a}\cdot\boldsymbol{b}$ over the computational domain; $\boldsymbol{f}$ is a vector that vanishes except that its streamwise component takes unit value in the region covered by the actuator disc; $\boldsymbol{u}$, p and Re are velocity, pressure and Reynolds number, respectivley.

Decomposing the flow field as the sum of a steady unperturbed flow and a perturbation flow i.e. $(\boldsymbol{u},p) = (\boldsymbol{U},P) + (\boldsymbol{u}',p')$, the optimal inflow perturbation can be defined as the one maximises the energy of the perturbation induced flow. The gradient of this final energy with respect to the inflow noise can be obtained by integrating an adjoint equation:

$$\partial_t \boldsymbol{u}^* + \boldsymbol{u}\cdot\boldsymbol{\nabla}\boldsymbol{u}^* - \boldsymbol{\nabla}\boldsymbol{u}\cdot\boldsymbol{u}^* - \boldsymbol{\nabla} p^* + Re^{-1}\nabla^2\boldsymbol{u}^* - \frac{2\boldsymbol{f}\cdot\boldsymbol{u}(\boldsymbol{f},\boldsymbol{u}^*)}{3(\boldsymbol{f},\boldsymbol{f})^2}\boldsymbol{f} = 0, \quad \text{with} \quad \boldsymbol{\nabla}\cdot\boldsymbol{u}^* = 0 \tag{2}$$

Clearly the full developing history of $\boldsymbol{u}$ at each time step is required.

Results

The flow downstream of a disc model, which is centred at the origin point with unit radius and streamwise extension 0.02, optimally perturbed by an inflow perturbation with turbulence intensity 0.01, is presented in figure 1. It is seen that the wake flow is significantly distorted owing to the amplification of upstream perturbations. The wake flow structure contains more than one non-zero azimuthal waves, manifesting the nonlinear growth of the perturbations. This nonlinearity is further indicated by the distribution of optimal inflow perturbation, shown in figure 2. It is also worth noting that the perturbation is concentrated in the streamwise and vertical components, with the azimuthal component much smaller.

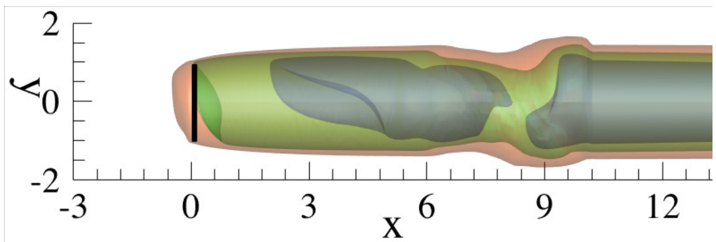

Figure 1. Iso-surface of streamwise velocity 0.3, 0.5, 0.8 (normalised by free-stream velocity) of the optimally perturbed flow. The optimal perturbation is introduced from the inflow boundary located at $x = -3$; the location of the disc model is indicated by the thick line.

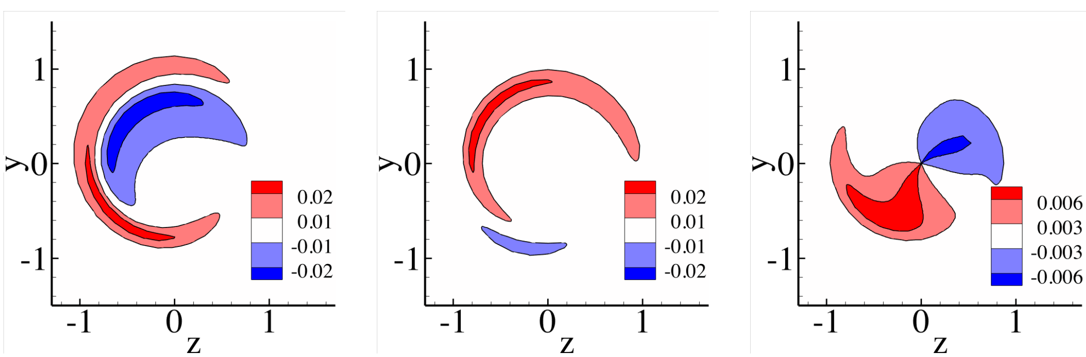

Figure 2. Components of the optimal inflow perturbation in streamwise, vertical and azimuthal directions from left to right.

References

[1] L. Chamorro and F. Porté-Agel. Effects of thermal stability and incoming boundary-layer flow characteristics on wind-turbine wakes: A wind-tunnel study. *Boundary-Layer Meteorol*, 136:515–533, 2010.

[2] X. Mao, H. M. Blackburn, and S. J. Sherwin. Calculation of global optimal initial and boundary perturbations for the linearised incompressible Navier–Stokes equations. *J. Comput. Phys.*, 235:258–273, 2013.