

# Towards roughness-based drag reduction

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Long Abstract

## Introduction

The final version of this paper will discuss the latest conclusions from our ongoing research on drag reduction in boundary-layer flows. We are motivated by the implication that reduced drag corresponds to energy savings. Our focus is on the relatively recently confirmed concept of reducing drag by means of appropriate surface roughness structures. The rationale for the research is that our recent theoretical results [1] suggest a new strategy for optimizing surface structures to delay transition and achieve energetically beneficial effects.

## 1. Boundary layers with a cross-flow component

Our recent results [1] are concerned with incompressible, fully three-dimensional boundary layers with a cross-flow component. The concept of cross-flow is well known and is often studied theoretically by the canonical von Kármán flow over a rotating disk. Boundary layers of this type are not only established over rotating disks, but they exist in very similar form, for instance, on highly swept wings or on many types of rotating machinery [2–4]. It is the cross-flow component with its inflection point that induces specific laminar-turbulent transition characteristics in this type of boundary-layer flow [2–5]. The consequence being that the flow, and transition, in all such boundary layers closely resemble each other and progress can be made using the model rotating-disk flow, as we do here.

The laminar-turbulent transition process of boundary layers with a cross-flow component is associated with two characteristic convective instability modes. One mode, commonly referred to as the Type I mode, is an inviscid instability that arises as a consequence of the inflection point (*cf.* Rayleigh Inflection point criteria, see e.g. [5]) on the cross-flow velocity profile. The other mode, commonly referred to as the Type II mode or the streamline-curvature instability, is a viscous instability that disappears in stability calculations when the viscous terms are excluded. Our ongoing studies focus on stationary disturbances since these are known to be naturally excited by even the most minute roughness protrusions [4] and they are the relevant modes resulting in transition in typical engineering applications. See [1] for further details regarding these stationary modes, as well as comments on non-stationary modes and absolute instability.

## 2. Leading-order surface-roughness effects

Generally the Type I mode by far dominates over the Type II mode. That is, the Type I mode occurs at lower Reynolds numbers and has substantially larger growth rates than the Type II mode [1, 4, 6]. However, our results show that the critical Reynolds numbers and the growth rates of both modes are substantially affected by surface roughness.

A key result of [1] is that increasing levels of the type of roughness considered there (i.e. that obtained from the *partial-slip* model due to [7]) substantially postpone the onset of the usually dominant Type I instability mode to higher Reynolds numbers; this is a desired result required for energetically beneficial transition delay. While the development of the secondary Type II mode is concurrently

advanced to *lower* Reynolds numbers, a further key result is that our study shows that the Type II mode never takes over from the Type I mode as the dominant instability mode responsible for transition. This same conclusion is reached using the alternative *surface-geometry model* of roughness due to [8]. Our results therefore appear to suggest is that the emergence of the Type II mode with increasing roughness halts the beneficial roughness effects on the Type I mode.

### 3. Energy analyses and subtleties between alternative roughness models

Our results have shown that for both the Type I and Type II instability modes, the main contributors to the energy balance are the energy production by the Reynolds stresses and conventional dissipation. For the Type I mode, dissipation *increases* while Reynolds-stress energy production *decreases* with the roughness level. Both these trends are beneficial since they imply a stabilization of the Type I mode by increasing roughness levels. These Type I results have been found to be independent of the particular roughness model tested.

However, further study has shown that, for the Type II mode, there is a qualitative difference in the results obtained for the energetically beneficial dissipation for the two different roughness models. For the Type II mode, energy dissipation *decreases* with the roughness level in the surface-geometry model, whereas it *increases* under the partial-slip model. This implies that Type II dissipation is sensitive to the roughness-related steady-flow base profile. Consequently, maximising dissipation by an appropriate surface-roughness pattern, that leads to the energetically optimal base profile, can theoretically lead to boundary-layer-transition delay and drag reduction. The crucial practical implication of this result is that the '*right type of roughness*' [9] has to be carefully designed to balance the particular effects of roughness on both mode types.

### References

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