Quantifying non-Newtonian effects in the context of transitional rotating boundary-layer flows

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Long Abstract

Introduction

The stability and transition of the boundary-layer on a rotating disk is a classical fluid mechanics problem that has attracted a great deal of attention from numerous authors over many decades. In the Newtonian limit there exists a vast wealth of material concerning the solutions of the rotating disk equations. Gregory, Stuart & Walker [1] were the first to investigate the stability characteristics of this boundary-layer flow, observing spiral modes of instability in the form of co-rotating vortices. Indeed, research on the von Kármán boundary-layer remains a topic of active study, the interested reader is referred to the recent review article by Lingwood & Alfredsson [2]. Flows of this nature have practical relevance to turbo-machinery where non-Newtonian fluids are commonplace. However, far less attention has been given to the corresponding non-Newtonian rotating disk problem. Very recently Griffiths [3] determined the base flow profiles for numerous generalised Newtonian fluid models. Using these results we investigate the convective instability of the boundary-layer on a rotating disk for shear-thinning power-law and Carreau fluids. Results show that accurately modelling the variation of viscosity within the boundary-layer is of paramount importance.

1. Formulation

After suitable non-dimensionalisation the base flow profiles, for power-law and Carreau fluids, are determined via a modification of von Kármán's classical similarity solution. The radial, azimuthal and axial velocities are denoted by U, V and W, respectively. The viscosity functions are defined in the following ways

$$\text{Power-law} \quad - \quad \mu = [(U')^2 + (V')^2]^{(n-1)/2}, \tag{1a}$$

$$\text{Carreau} \quad - \quad \mu = 1 + c_0\{1 + k^2[(U')^2 + (V')^2]\}^{(n-1)/2}, \tag{1b}$$

where the primes denote differentiation with respect to η (the similarity coordinate), n is the fluid index, c_0 is the viscosity ratio and k is the dimensionless form of the relaxation parameter. Throughout this investigation c_0 and k are held fixed. We find that, for shear-thinning power-law fluids, the viscosity function is unbounded within the confines of the boundary-layer, this is because U' and V' are decaying to zero far from the disk whilst the fluid index is in the range $0 < n < 1$. However, the shear-thinning Carreau model predicts a finite value of viscosity at the boundary-layer edge, this being $\mu = 1 + c_0$. An in-depth discussion of these, and other flow profiles can be found in Griffiths [3].

2. Convective stability analysis

The stability analysis, applied at a specific radius, involves imposing infinitesimally small disturbances on the steady mean flow, in the form of scaled normal-mode quantities

$$(u, v, w, p) = (\hat{u}, \hat{v}, \hat{w}, \hat{p})(\eta; \alpha, \beta, \omega; R, n)e^{\mathrm{i}(\alpha r + \beta\theta - \omega t)}. \tag{2}$$

The frequency of the disturbance in the rotating frame is ω (taken to be zero in this stationary study), the complex radial wavenumber is $\alpha = \alpha_r + \alpha_i$ and β is the real azimuthal wavenumber. After making

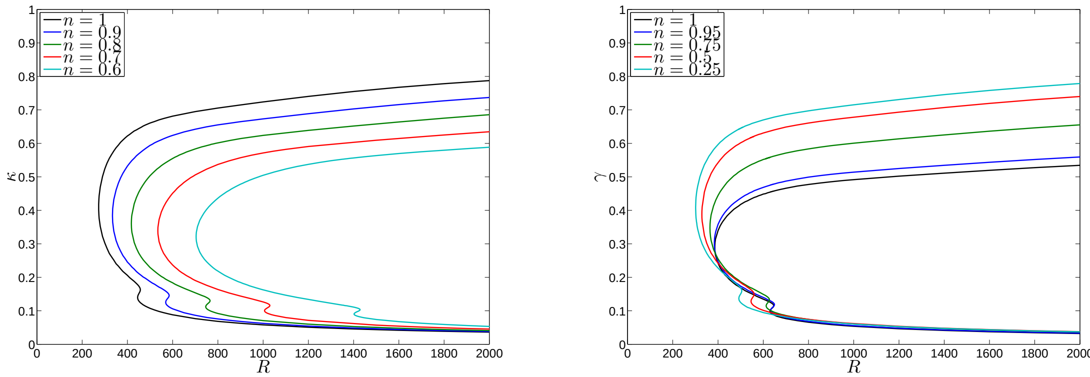

Figure 1. Neutral stability curves for shear-thinning power-law fluids (left) and Carreau fluids (right).

necessary assumptions the stability equations may be written as a set of six first-order ODEs using the transformed perturbing variables $\phi_i(\eta)$ with $i = 1, 2 \dots 6$. The governing equations for power-law fluids can be found in Griffiths *et al.* [4]. The eigenvalue problem defined by the stability equations is solved with the homogeneous boundary conditions $\phi_i = 0$ at $\eta = 0$ and $\phi_i \to 0$ as $\eta \to \infty$, for all i. This eigenvalue problem is solved for certain combinations of values of α, β and ω at each Reynolds number, R, and for the specified value of n. From these we form the dispersion relation, $D(\alpha, \beta, \omega; R, n) = 0$, at each n, with the aim of studying the convective instabilities. In order to investigate the structure of the spatial branches at each n, we solve the dispersion relation for α whilst marching through values of β at fixed R. For each n in the particular range of interest two spatial branches determine the convective instability characteristics of the system. Neutral curves, defined by $\alpha_i = 0$, have been calculated for shear-thinning power-law and Carreau fluids, see figure 1.

3. Discussion

The power-law model predicts a strong stabilising effect as the fluid index is decreased, whilst the Carreau model suggests that shear-thinning destabilises the boundary-layer flow. These rather striking results are attributed to the inability of the power-law model to describe shear-thinning flows for vanishing shear-rates, i.e. far from the disk surface. These results clearly show the importance of accurately modelling the variation of viscosity within the boundary-layer. The power-law model may be useful for describing experimental results in regions of moderate shear-rate, however, it fails within this theoretical framework. As such, we conclude that the Carreau model provides a better physical representation of the boundary-layer flow and hence that introduction of shear-thinning fluids will have a destabilising effect.

References

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