Error Propagation Dynamics of PIV-based Pressure Field Calculations

Zhao Pan, Department of Mechanical Engineering, Brigham Young University, Provo, UT, USA

Jared, Mathematical Department, Brigham Young University, Provo, UT, USA

Tadd Truscott, Department of Mechanical and Aerospace Engineering, Utah State University, UT, USA

Long Abstract

Introduction

After more than 20 years of development, PIV has become a standard non-invasive velocity field measurement technique [1], and promises to make PIV-based pressure calculations possible [2]. However, there are always uncertainties in PIV velocity fields, and these uncertainties will eventuality propagate through integration and contaminate the pressure field. Some researchers have noticed this issue, and several techniques have been developed to reduce these errors [3]. For example, one type of method averages many pressure calculations along different integral paths by taking advantage of the scalar property of the pressure field. Namely, the integrated pressure value at arbitrary locations in the flow field is independent of the integral path. Baur and Kongeter [4] directly integrated the simplified Navier-Stokes equations in an explicit scheme. They utilized time-resolved PIV data to determine the turbulent flow passing over a wall. At each nodal point, four integrals were calculated from neighboring nodes and averaged to formulate the pressure estimation. Liu and Katz [3] proposed an omni-directional integration scheme to solve the Poisson pressure equation from a virtual boundary outside the flow field. This method was validated by a synthetic flow and then applied to a cavity flow. Dabiri et al. [5] proposed an algorithm that used the median of the pressure calculated by integrating pressure gradient along eight paths as the estimation of local pressure at each nodal point in the field (Dirichlet boundary conditions were employed on the edges of the flow field). A temporal filter was utilized to reduce the uncertainties in the velocity field from the PIV. This approach was applied to the flow around free swimmers (e.g., jellyfish and lamprey) with impressive success. In a more recent paper, Charonko et al. [6] reviewed and evaluated different factors (e.g., resolution of PIV data, types of flow, and velocity field smoothing functions) of the calculation scheme used in PIV-based pressure acquisition. The authors report that the Poisson solvers are sensitive to aforementioned factors. Error can vary from less than 1% to more than 100% and the pressure calculation is highly dependent on the flow type, which implies that there is no universally optimized method for all flow types. One underlying piece of this puzzle still remains to be unraveled: the underlying mechanisms of noise accumulation from the PIV measurements to the pressure estimations.

1. Methods

In this research, we propose analysis of the uncertainty propagation dynamics of the PIV-based pressure field calculation. Our analysis shows how the uncertainties in the velocity field ($\boldsymbol{u}$) propagate to the pressure field (p) through the Poisson equation, $\nabla^2 p = f(\boldsymbol{u})$, which is a popular calculation principle. First of all, we model the dynamics of error propagation by boundary value problems (PVBs):

$$
\begin{aligned}
&PDE: \quad \nabla^2 e_p = f(\boldsymbol{e}_u) \quad in \quad \Omega \\
&Dirichlet\ BCs: \quad e_p = h(\boldsymbol{u}) \quad on \quad \partial\Omega \\
&Neumann\ BCs: \quad \nabla(e_p) = g(\boldsymbol{e}_u)\cdot \boldsymbol{n} \quad on \quad \partial\Omega
\end{aligned} \tag{1}
$$

Where, e_p and $\boldsymbol{e_u}$ are error in pressure and velocity field, respectively; $f(\cdot)$, $g(\cdot)$, and $h(\cdot)$ are functions of errors; Ω is the domain of flow field. Next, L^2-norm and/or L^∞-norm is utilized as the measure of error level of the velocity and pressure field ($||\boldsymbol{e_u}||_{(\Omega)}$ and $||e_p||_{(\Omega)}$, respectively). Finally, with some analysis tools such as maximum principle, Poincare inequality and Agmon's inequality, the error level of pressure field can be bounded by the error level of the data of the BVPs ($||g(\boldsymbol{e}_u)||_{(\Omega)}$ in field, as well as $||f(\boldsymbol{u})||_{(\partial\Omega)}$ and $||h(\boldsymbol{u})||_{(\partial\Omega)}$ on boundary), by considering the well-posedness of the BVPs. Specifically, we exam if and how the error in the pressure field (e_p), depend continually on the data of BVPs ($f(\boldsymbol{e}_u)$, etc.). From a physics standpoint, factors such as the geometry of the flow field, boundary conditions, and velocity field noise levels will be discussed analytically. The results are inherently independent of the numerical methods from the pressure field solver. Synthetic flow fields will be used to validate the results.

Our preliminary analysis mathematically supports the claims of [6] that the uncertainty level of the velocity field, temporal and spatial resolution of the PIV data, boundary condition type, and flow pattern all impact the error propagation; and implies that geometry and scale of the flow field are also important factors. As an example, figure 1 presents the error level of the pressure field versus length scale of the flow field when different boundary conditions (Dirichlet and Neumann) are applied. The curves indicate the highest possible error level in the pressure field with the same level of uncertainties in data as input into the Poisson solver. The results form this research may shed light into methods of optimizing experimental set-up and benefit from improved physics-based error reduction techniques.

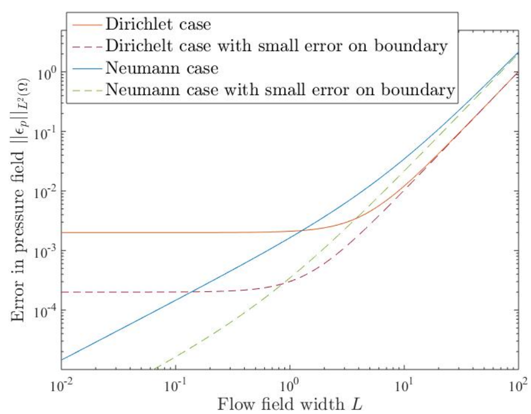

Figure 1. Bounds of the L^2-norm of error in a pressure field. The red and blue curves indicate the highest possible uncertainty level in the pressure field with the same level of error introduced in the data (2^{-3} on boundary and in field) for Dirichlet and Neumann cases, respectively. The purple and green dashed lines show Dirichlet and Neumann case with the same uncertainty level as other cases in the flow field, but with less error (2^{-4}) on the boundaries.

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