Simplified Modeling of Cavitating Flow with Thermodynamic Effect for Homogeneous Model

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Long Abstract

Introduction

It is well known that the thermodynamic effect has significant influence on the suppression of cavity volume. The thermodynamic effect appears in cryogenic fluid, water at high temperature and thermal sensitive liquid. In the liquids, when cavitation occurs, latent heat of evaporation is deprived, local temperature decreases, saturated vapor pressure decreases and then cavitation is suppressed. In numerical simulation, however, most of cavitation model was developed for liquid at room temperature, where the influence of thermodynamics is small and ussualy neglected because the temperature decrease by evaporation is very small [1]. Additionally, in homogeneous model for cavitating flow in which gas - liquid interface is coarse grained, temperature decrease in the vicinity of gas-liquid interface can not be reproduced. Hence, in this study, four kinds of thermal models for homogeneous model are compared: the simplify energy equation such as the model without Latent heat effect (the equilibrium model) and the Latent heat model, Iga's simplify thermodymics model [2] and Tsuda's thermodynamic model [3] based on B-factor. At first, the 1D simulation of converging – diverging nozzle with hot water experiment by Abuaf, et al. [4] is validated. Then, the 2D simulation of quarter hydrofoil for cryogenic cavitation by Hord [5] is considered for validation.

1. Numerical Methods

Locally homogeneous and compressible gas-liquid two-phase medium model. In this study, cavitating flow is simulated based on the in-house compressible homogeneous model code [2]. By assuming local equilibrium in pressure, temperature and non-slip velocity between gas and liquid phase, the governing equations for gas-liquid two phase medium can be written in a simple form as pseudo - single phase medium:

$$\frac{\partial \mathbf{Q}}{\partial t}+\frac{\partial \mathbf{E}-\mathbf{E}_{\mathbf{v}}}{\partial x}+\frac{\partial \mathbf{F}-\mathbf{F}_{\mathbf{v}}}{\partial y}=\mathbf{S}$$

$$\mathbf{Q}=\left[\rho,\rho u,\rho v,\rho Y\right]^T,\mathbf{E}=\left[\rho u,\rho u^2+p,\rho uv,\rho uY\right]^T,\mathbf{F}=\left[\rho u,\rho uv,\rho v^2+p,\rho vY\right]^T \qquad (1)$$

$$\mathbf{E}_{\mathbf{v}}=\left[0,\tau_{xx},\tau_{xy},0\right]^T,\mathbf{F}_{\mathbf{v}}=\left[0,\tau_{xy},\tau_{yy},0\right]^T,\mathbf{S}=\left[0,0,0,m\right]^T$$

In that, ρ, u, v, p, Y are the density, velocity in x and y direction, pressure and mass fraction of gas phase, respectively. m is mass change rate by cavitation. The liquid phase is assumed as compressible and the density is expressed by Tammam equation. The gas phase is considered as ideal-gas. Then, the equation of state for mixture phase is derived as:

$$\rho=\frac{p(p+p_{\mathrm{c}})}{K_1(1-Y)p(T+T_{\mathrm{c}})+R_{\mathrm{g}}Y(p+p_{\mathrm{c}})T} \qquad (2)$$

Present thermodynamics model. In present study, instead of solving equation for total energy [2], the simplified model in form of conservation equation for temperature is solved, in that the viscous term and thermal conduction term are neglected [6]. For the thermal equilibrium model, the Eq. (3) is solved without the Latent heat of phase change, L. And the Latent heat model, the L is added in order to account the heat lose by evaporation and heat input by condensation.

$$C_p\left(\frac{\partial \rho T}{\partial t}+\frac{\partial \rho u T}{\partial x}+\frac{\partial \rho v T}{\partial y}\right)=\frac{Dp}{Dt}+\left(C_{p\mathrm{l}}-C_{p\mathrm{g}}\right)Tm+Lm \qquad (3)$$

In that, C_p is isobaric specific heat and subscript g and l denote gas and liquid phase. In calculation, temperature of mixture phase T is derived from the above equation.

Cavitation model. In this study, the Merkle cavitation model [7] and the model suggested by Iga, et al.[2] are chosen for validation. Saturated vapor pressure, p_v, which is threshold of evaporation and condensation in phase change term m in Eq. (1), is estimated by two ways. In thermal equilibrium model, the Latent heat model and Iga's thermodynamics mode, p_v is calculated by Eq. (4) using equation of saturated vapor pressure by Sugawara [8] with local T estimated from Eq. (3).

$$p_v = p_0 \exp\left[\left(1-\frac{T_0}{T}\right)\left(a_1+\left(a_2+a_3 T\right)\left(T-a_4\right)^2\right)\right] \quad (4)$$

In Tsuda's thermodynamic model [3], temperature decrease is estimated by using B-factor. Then the vapor pressure p_v is calculated as:

$$p_v{}^* = p_v - \frac{\alpha}{1-\alpha}\frac{\rho_g L}{\rho_l C_{pl}} G_{sat} \ , \quad (5)$$

Numerical and Validation. In this study, the fluid is assumed as inviscid. The governing equations are solved by finite difference method. The explicit TVD MacCormack scheme is used, which has second order accuracy in time and space. The present method has been validated for 1D multiphase shock tube problem [9] and 2D axisymmetric hemispherical headform [10]. Figure 1 shows the comparison result of three models: thermal equilibrium model (without notation), the Latent heat model (LH) and Iga's thermodynamics mode in 1D converging - diverging nozzle at T = 421K, where the throat locates at x = 0.3m. In that, the Latent heat model shows the improvement in prediction of pressure distribution at the diverging part region behind the noozle throat comparing to other models. However, the over estimation of vapor void fraction resulted in all models. Figure 2 is the comparison results of four models in case T = 422.5K. In that, although the higher vapor void fraction than experiment observed by all four cases, the one estimated by Iga's model and Latent heat model have similar tendency comparing to the experiment. Again, the pressure prediction by the Latent heat model closes to measure data. Besides, there is no difference between the results by Tsuda's model and thermal equilibrium model.

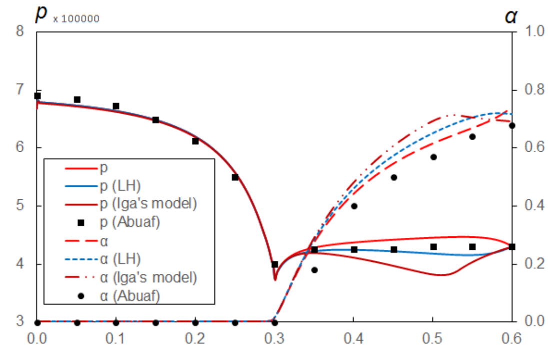

***Fig.1** Comparison of pressure and void fraction along nozzle three models and Abuaf's results in case T=421K*

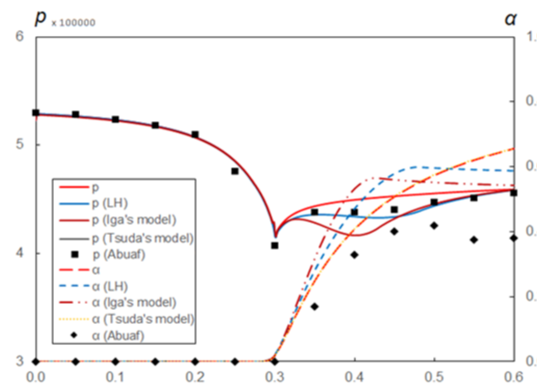

***Fig.2** Comparison of pressure and void fraction along nozzle three models and Abuaf's results in case T=422.5K*

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