



[Extended Abstract]

A conditioning technique for projection-based reduced order models

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Introduction

Reduced order models (ROMs) are important in several fields of science and engineering, and in particular in turbomachinery, due to the need to use high fidelity finite element models whose computational burden can be extremely high [1, 2]. In particular, several studies focus on the numerical stability of ROMs [3, 4, 5], which is a fundamental criterion to distinguish between feasible and unfeasible model reduction solutions. This study focuses on the numerical issues associated with forming ROMs using a set of basis vectors that are theoretically linearly independent, but not numerically. In such cases, numerical issues can result in inaccurate or even unstable ROMs. A new conditioning approach is introduced to solve this challenge by using only transformations in the reduced order space, and thus not requiring heavy computations. The manipulation of the left and right basis vectors is carried out implicitly, without prior knowledge of the transformation matrix and without tuning. The approach is particularly relevant because it can be applied even in those cases where tuning is not possible because the solution of the full order model is not available.

Methods

An approach based on singular value decomposition is used here to enforce numerical stability and to improve the accuracy of the ROM. The process starts with forming a ROM using standard approaches. Next, this ROM is conditioned. The conditioned ROM is obtained by projection onto a secondary reduced set of coordinates corresponding to a set of right-singular vectors of the reduced order mass matrix. The secondary projection vectors are chosen based on the corresponding singular values. The key idea is that small singular values correspond to an ill conditioned or singular mass matrix. The corresponding secondary projection vectors are iteratively discarded until a stability criterion is reached. After the conditioning procedure is complete, a conditioned ROM is obtained with structural characteristics similar to the ones of the unconditioned system. A set of metrics is also introduced in this work to evaluate the error due to conditioning. The proposed conditioning technique is optimal in the sense that, given the original transform, it provides the most accurate possible ROM that can be built using the initial set of basis vectors.

Results

The method has been applied to the complex structure shown in Figure 1. The results shown in Figure 2 reveal that the conditioned results are substantially more accurate. Not only the unconditioned case

is much worse in terms of eigenvalue precision, but it also has several positive eigenvalues, which indicate that the unconditioned ROM is erroneously unstable.

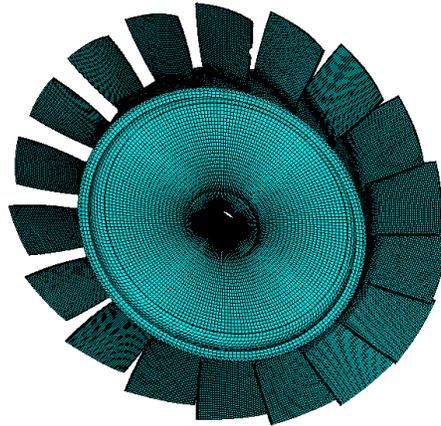


Figure 1. Full order model of a bladed disk with blends

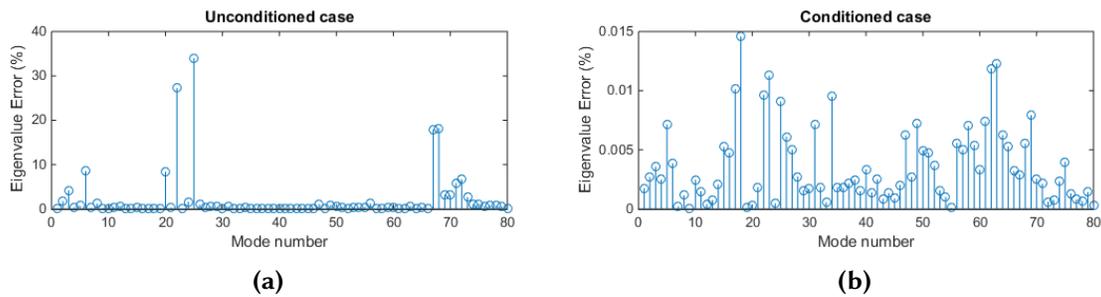


Figure 2. Eigenvalue errors between conditioned and unconditioned ROMs and the full order model shown in Figure 1

References

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