A Novel Approach for the Experimental Nonlinear Modal Analysis of Shrouded Turbine Blades

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Introduction

In the nonlinear structural analysis of turbine blades, experiments are indispensable for model validation and identification purposes. Nonlinear modes are considered useful in describing the vibration signature of nonlinear mechanical systems and have been applied successfully to turbomachinery systems. Yet, experimental techniques for the nonlinear modal analysis, which are designed for the underlying conservative system in the presence of weak damping, cannot be applied to systems with friction. Conversely, nonlinear damping identification methods based on vibration decay measurements often suffer from a coarse resolution due to the limited measurement time.

For the modal analysis of structures with nonlinear damping, the extended periodic motion concept as defined in [1] examines self-excited steady-state periodic motions. Consider an autonomous dynamical system with the mass and stiffness matrices \( M = M^T > 0 \) and \( K = K^T > 0 \), where the nonlinear restoring and damping forces are gathered in \( \tilde{g}(x, \dot{x}) \) with the vector of generalized coordinates \( x \). To overcome the typically decaying motion, a mass-proportional negative damping term \( -\xi M \dot{x} \) is introduced,

\[
M \ddot{x} + K x + \tilde{g}(x, \dot{x}) - \xi M \dot{x} = 0.
\] (1)

The nonlinear modes are defined as the periodic motions of the autonomous surrogate system (1). To achieve periodicity, the artificial term must be large enough to compensate the system’s dissipation.

The herein proposed method for the experimental nonlinear modal analysis of dissipative structures investigates modes according to the extended periodic motion concept based on experimental data only. It has been tested previously by the authors with the aid of an academic example, namely a numerical model of a steel beam with one elastic Coulomb element. In this contribution, it is shown that the method is also applicable to structures consisting of multiple components coupled by friction joints. To this end, a system of three blades with shroud joints is considered. In this work, a numerical validation of the experimental approach is carried out.

Experimental Approach and Validation

For the experimental implementation, the negative damping term in Eq. (1) is interpreted as the excitation force. Since the mass matrix is unknown in an experiment, the excitation is replaced by a phase resonant harmonic excitation at a finite number of points. The isolation of the mode by this imperfect excitation can be evaluated based on power quantities analogously to [2].
The excitation is implemented by means of a feedback controller, i.e., a phase-locked-loop. By incrementally increasing the force level, the backbone curve is tracked and the eigenfrequencies $\tilde{\omega}_0$ and deflection shapes can be extracted as a function of the vibration level [2]. If the mode is isolated perfectly, the dissipated power is equal to the power of the excitation. Therefore, a nonlinear damping coefficient $\tilde{\delta}$ can be defined and computed with

$$P_{\text{exc}} = \sum_k \frac{1}{2} f_k v_k \cos(\varphi_k) = \tilde{\delta}(q) \tilde{\omega}_0^3(q) q^2 = P_{\text{diss}}. \quad (2)$$

Thus, the eigenfrequency and fundamental harmonic magnitudes of the excitation force $f$ and reference velocity $v$ including the phase angle $\varphi$ for each excitation point $k$ are needed. The modal amplitude $q$ can be derived from the mass-normalized linear deflection shape vectors obtained with a standard linear experimental modal analysis.

To test the method, a model of three blades is used which are clamped at the root and circumferentially arranged. The friction contact at the shroud is modeled by elastic Coulomb elements at four contact surfaces, each consisting of two relative degrees of freedom in tangential direction under constant normal force and constant friction coefficient. The structure is excited at the midspan of the center blade’s leading edge in axial direction.

Figure 1 shows the results of the simulated measurements compared to reference values calculated with the method proposed in [1]. The overall behavior of the system is captured well. Apparently, the deviations in the frequency-energy dependence are more significant than those in the damping-energy dependence. A potential explanation for this is that the vibration response of friction-damped systems is comparatively frequency-insensitive near resonance, making it more difficult to capture the precise location of the resonance. It is assumed that the nonlinear mode isolation could be improved by selecting a different exciter location or increasing the number of excitation points. Hence, the ongoing study focuses on the choice of excitation and measurement points as well as on the influence of the shaker-structure interaction.

![Figure 1](image)

**Figure 1.** Natural frequency (left) and the modal damping ratio (right) as a function of the vibration level. The frequencies and displacements are normalized to the full stick linear case.

**References**
