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[Extended Abstract]

Rotor–stator interaction: reduction of a nonsmooth thermo-elastic unidimensional model

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Introduction

In aircraft engines, rotor–stator contact interactions can be described by including unilateral constraints in the model, preventing interpenetration between blade and casing. The simplest approach to simulate such a nonsmooth model is to regularize contact, however doing so introduces numerical drawbacks such as numerical stiffness requiring artificially small time steps, low local order of consistency, low global order of accuracy, stability issues [1]. Another possibility is to use Carpenter's numerical scheme [2], consisting in a prediction step to detect interpenetration, and a projection step to statisfy the unilateral constraint if needed. Though sufficient in many cases, this method does not rely on strong mathematical settlements and faces stability issues in number of contexts. Additionally, it hides the impact law, offering no control on it.

This contribution investigates a unidimensional blade impacting a rigid body whose dynamics is formulated as non-smooth thermo-elastic model formulated as a Measure Differential Inclusion [1] solved using dedicated non-smooth numerical schemes. The thermo-elastic model is then reduced using a Craig-Bampton-like model reduction technique to investigate the scalability of such methods to industrial problems involving a large number of degrees of freedom.

1. Formulation of the thermoelastic model

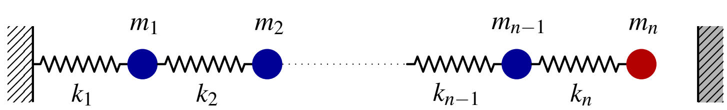

Figure 1. Unidimensional blade–casing thermoelastic model.

For simplicity, the spring–mass system of Fig. 1 is considered. The nth mass, representing the tip of the blade, undergoes a contact condition with an obstacle, corresponding to the carter. Its thermoelastic dynamic behaviour is described (in a not strictly rigourous way) by

$$\begin{cases} M\ddot{x} + C_1\dot{x} + K_1 x + K_{12}\theta = f_{\text{contact}} + f_{\text{struc}} & \text{(1a)} \\ C_2\dot{\theta} + K_2\theta = f_{\text{therm}} + f_{\text{tip}} & \text{(1b)} \end{cases}$$

where x is the vector of displacements, θ the vector of temperatures, M, C_1 and K_1 are the mass, stiffness and damping matrices, C_2 and K_2 are the heat capacity and heat conductivity matrices, f_{contact}, f_{struc}, f_{therm} and f_{tip} are the contact forces, structural loading, heat loading and the heat production at the blade tip due to contact. The contact acts only on the last node: $f_{\text{contact}} = \lambda e_n$ where λ is the normal contact force satisfying the Signorini conditions $\lambda \geq 0$, $x - d \leq 0$ and $\lambda(x-d) = 0$ meaning that there is no sticking, no interpenetration and the normal contact force can be non-zero only when the gap is closed. A Newton impact law is chosen: $\dot{x}_n^+ = -e\dot{x}_n^-$. Altogether, these conditions can be written with a Measure Differential Inclusion (MDI): $\lambda \in \partial\psi_{T_{\mathbb{R}^+}(x-d)}\big((v^+ + ev^-)/(1+e)\big)$. where $\partial\psi_{T^+_{\mathbb{R}}}$ is the subderivative of the indicator function of the tangent cone to $\mathbb{R}^+$ evaluated in $x - d$. Contact is assumed to create a heat flux on the last node accounting for heating due to dry friction, proportional to the normal contact force λ: $f_{\text{tip}} = \alpha\lambda$.

The MDI is time-discretized using a prox operator [1] and an implicit Euler numerical scheme.

2. Model reduction

A variation of Craig-Bamptom reduction for thermo-elastic systems has been proposed [3], based on a second order differential equation gathering both dynamic and heat equations, with a singular mass matrix. Here, a similar reduction is applied to the system written in the first-order form, i.e. for a state variable $X^\top = [x, v, \theta]$. In very short, the reduction consists in keeping the contacting node as such (*boundary node*) and modelling the behaviour of the other nodes (*interior nodes*) using a few low-eigenvalue modes.

3. Results

Model parameters are tuned to match a typical blade of a centrifugal compressor impeller using $n = 20$. The contacting node is subjected to a periodic sinusoidal loading. Results are shown in Fig 2 for a time step of 10^{-4} s and a coefficient of restitution of $e = 0$. Contact transitions are very clear and the simulation is very robust: increasing the time step results in decreasing accuracy but not in stability issues. The heat creation during contact phases results in thermal expansion (gap reduction) and increasing contact force after each contact. Results are well approximated using the reduction with only three mechanical modes and twelve thermal modes.

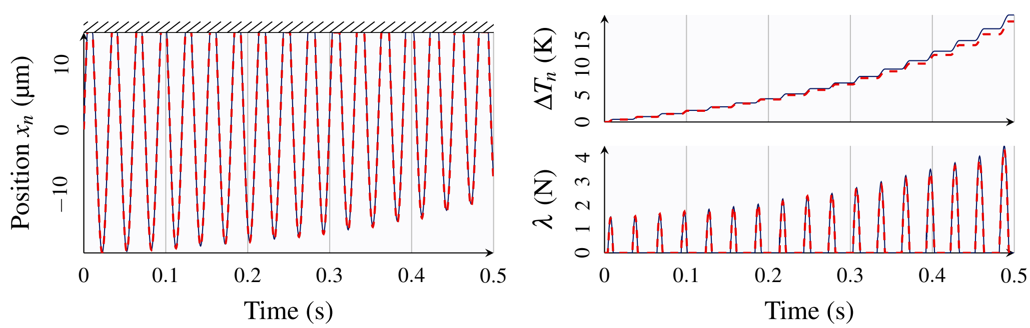

Figure 2. Response of the blade tip to a periodic loading. *Left* Displacement of contacting node. *Right, top* Variation of contacting node temperature. *Right, bottom* Contact force. [—] Full model. [- - -] Reduced Model.

References

[1] V. Acary and B. Brogliato. *Numerical methods for nonsmooth dynamical systems: applications in mechanics and electronics.* Springer Science & Business Media, 2008.

[2] N. Carpenter, R. Taylor, and M. Katona. Lagrange constraints for transient finite element surface contact. *International journal for numerical methods in engineering*, 32(1):103–128, 1991.

[3] P. Nachtergaele, D. Rixen, and A. Steenhoek. Efficient weakly coupled projection basis for the reduction of thermo-mechanical models. *Journal of computational and applied mathematics*, 234(7):2272–2278, 2010.