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[Extended Abstract]

Spatio-temporal Koopman Decomposition in cross-flow wind turbines

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The main goal of this work is to study the spatio-temporal structures found in the wake of an offshore wind turbine. For this purpose a new method, called Spatio-Temporal Koopman Decomposition (STKD), will be used. Traveling waves found in the wake of this turbine will be studied in depth, shedding light on the complex phenomena behind this type of flows.

Introduction

The recent knowledge of environmental problems, such as the green-house effect or the hole in the ozone layer, yields the need to increase the use of renewable energies instead of fossil fuels, to produce several types of energy. Offshore wind energy is a type of renewable energy that uses wind power to generate electricity. Due to its good performance, this type of energy has become very popular during the last years. Offshore wind speeds availables are higher than the ones found in land, making this energy more efficient than others and consequently, making profitable the construction of wind farms offshore.

Due to its high complexity, the deep study and understanding of the complex behaviour that takes place in the wake of an offshore wind turbine is a research topic of high interest. Wake interactions in wind farms may lead to highly complex non-linear phenomena that need to be analyzed using sophisticated techniques.

In this conference we will present the spatio-temporal analysis carried out in the wake of an offshore turbine. The analysis will be performed using a new technique, called Spatio-Temporal Koopman Decomposition (STKD)[2], that is based on a higher order dynamic mode decomposition (HODMD)[1]. This tecnique combines classical DMD [4] with Taken's delay embedding theorem [5] using time-lagged snapshots. The main benefit of STKD lies on its possibility of capturing the most relevant spatio-temporal structures and traveling waves, interacting in the wake of an offshore turbine.

1. Spatio-temporal Koopman Decomposition

The main goal of STKD is to decompose spatio-temporal data $u(x, y, t)$ as a sum of spatial and temporal DMD modes q_{mn} in the following way:

$$u(x, \boldsymbol{y}, t) \simeq u^{DMD}(x, \boldsymbol{y}, t) \equiv \sum_{m,n=1}^{M,N} a_m^x a_n^t q_{mn}(\boldsymbol{y})\, e^{(\nu_m + i\kappa_m)x + (\delta_n + i\omega_n)t} \tag{1}$$

where κ_m and ω_n, are the spatial wave numbers and temporal frequencies, respectively, ν_m and δ_n are the spatial and temporal growth rates and a_m^x and a_n^t are the spatial and temporal amplitudes. This method identifies traveling waves, whose phase velocity is defined as $c = \omega_n/\kappa_m$.

The STKD algorithm considers two main steps. As first step a higher order singular value decomposition (HOSVD) is performed in order to clean spatial redudancies, usualy found in data coming from numerical simulations, and noise, commonly found in experimental data. HOSVD applies a SVD in each one of the spatial directions. As second step a HODMD is applied in both, temporal and spatial data. HODMD is a technique that combines classical DMD with time-lagged snapshots, reducing not only data uncertainty, but also increasing their accuracy and covering a wider range of applications than DMD. The good performance of this technique has been tested in several cases presented in the literature in both numerical [1] and experimental data[3].More details about the method will be presented at the time of the conference.

References

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