Experimental Prediction of Instability in Rotor Seal Systems using Output Only Data

Christian Wagner*, Thomas Thümmel, Daniel Rixen

Abstract
In common high-speed turbomachinery or pumps, usually contactless seals are used to separate different fluids and pressure levels. The seal gap causes a fluid flow, which generates rotational speed dependent forces to the rotor in the deflectional and tangential directions. They act like cross-coupled stiffness and can excite the rotor system to large vibrations, the instability occurs similar to the 'oil whip' phenomenon in journal bearings. Thus, a diagnosis method is necessary to ensure safe operation. To predict the speed limit of the machinery, two methods are presented in this research: an analysis using active magnetic bearing excitation and an output only damping ratio estimation method. The methods are experimentally evaluated on a test rig and using numerical simulations.

Keywords
Rotordynamics, stability diagnosis, rotor seal system, active magnetic bearing

INTRODUCTION
Seals in turbines, centrifugal pumps or compressors are commonly used to separate different fluids and pressure levels. Because of the high rotational speeds of common turbomachines, contactless seals, such as floating ring, labyrinth or seal gaps are inserted between the rotating and the stationary parts. The clearance which always exists around these contactless seals permits fluid flow through the gap. For an eccentric rotor position, the fluid-velocity distribution inside the seal becomes asymmetric, which generates forces on the rotor. These can result in stiffening, restoring, and damping effects, and in particular tangential forces, caused by the whirling fluid flow inside the seal gap. These are responsible for cross-coupling terms in the stiffness matrix. They cause a rotor vibration at the onset speed of instability, leading to large displacements and to a breakdown of the system, similar to the ‘oil-whip’ phenomenon in journal bearings. Furthermore, the rotor-system or the seal condition can change during lifetime. Thus, an experimental diagnosis method is necessary to avoid rotor instability and to ensure safe and stable long-term operation.

A modern approach is to predict the stability limit of the entire system using output only run up measurements. Observing the parameters during normal operation, a change of the stability limit can be detected and used as an indicator for monitoring.

The first focus of this contribution is an introduction to rotor-seal system instability using a JEFFCOTT rotor model and rotordynamic seal coefficients, based on [1]. The second focus is the comparison of several experimental methods on the test rig with and without additional excitation using an active magnetic bearing (AMB). This offers possibilities for actual turbo-machinery applications.

In the recent decades, a lot of effort has gone into modeling and identifying the dynamics of rotor seal systems. Amongst the first researchers to model the effects of seals using rotordynamic seal coefficients were Black [2] and Childs [3], who solve the bulk-flow equations and create analytical short seal solutions. Muszynska [1],[4] improves the seal models and includes the eccentricity influence.

Other experimental investigations using AMB’s in rotor-seal systems are [5], [6] and [7]. Using a levitating rotor to examine the seal is performed by [8] and [9].

The idea of using output only data to identify the system’s damping ratio is also used in different ways by [10], [11] and [12]. A lot of investigations take place in the field of operational modal analysis. An interesting method to remove the unbalance response is proposed by [13].

The fundamentals of rotordynamics, modeling and simulation are clearly described in [14], [15] and [16].

1. MODELING
The rotor model used to describe the dynamic behavior of a rotor seal system is based on the JEFFCOTT or LAVAL rotor, see [14]. Based on the investigations of Childs[16] and Muszynska[15], the rotordynamic behavior of the seals is modeled using rotordynamic seal coefficients.
1.1 Rotor Model

The simplified rotor model (see fig. 1), consists of a flexible, massless shaft with a mass disk supported by two rigid bearings. Defining the rotor displacement as \([x \ y]^T\), the equation of motion can be written as follows:

\[
\begin{bmatrix}
    m_r & 0 \\
    0 & m_r
\end{bmatrix}
\begin{bmatrix}
    \ddot{x} \\
    \dot{y}
\end{bmatrix}
+ \begin{bmatrix}
    k_r & 0 \\
    0 & k_r
\end{bmatrix}
\begin{bmatrix}
    x \\
    y
\end{bmatrix} = \vec{h}
\]

where \(m_r = 5\) kg is the rotor mass and \(k_r = 0.293\) MN/m is the shaft stiffness. The equivalent forces \(\vec{h} = \vec{h}_u + \vec{h}_c + \vec{h}_s\), representing the unbalance force, external forces, seal forces and so forth, are used to couple different models (seals, bearings etc.) and excitations. Therefore the rotor’s first natural frequency is denoted by \(\omega_1 = \sqrt{\frac{k_r}{m_r}}\).

1.2 Rotor-Seal Model

The seal forces are modeled using the rotordynamic coefficients, according to \([16][15][14]\):

\[-\vec{h}_s = \begin{bmatrix}
    m_{xx} & 0 \\
    0 & m_{xx}
\end{bmatrix}
\begin{bmatrix}
    \ddot{x} \\
    \ddot{y}
\end{bmatrix}
+ \begin{bmatrix}
    c_{xx} & c_{xy} \\
    c_{yx} & c_{yy}
\end{bmatrix}
\begin{bmatrix}
    \dot{x} \\
    \dot{y}
\end{bmatrix}
+ \begin{bmatrix}
    k_{xx} & k_{xy} \\
    k_{yx} & k_{yy}
\end{bmatrix}
\begin{bmatrix}
    x \\
    y
\end{bmatrix} \equiv \vec{h}_s
\]

where \(m_{xx}, c_{xx}\) and \(k_{xx}\) are the seal coefficients of direct mass, damping and stiffness. \(c_{xy}\) and \(k_{xy}\) are the cross coupled damping and stiffness, same for the \(y\) direction, respectively. The coupled rotor seal minimal model is shown in fig. 2. The coefficients are numerically calculated using Muszynska’s model \([1],[4]\), with the parameters as in HuA \([18]\). The model is based on the bulk flow theory, which neglects fluid velocity and pressure differences in the radial direction. The solving process is as follows: The main axial velocity according to the rotational speed, Reynolds numbers, friction model and pressure difference is numerically calculated. Then the rotordynamic coefficients are derived from the fluid velocity distribution using the short seal solution for the centered rotor. Applied for a symmetrical seal, the seal force relation is, see \([4]\):

\[
\begin{bmatrix}
    m_s & 0 \\
    0 & m_s
\end{bmatrix}
\begin{bmatrix}
    \ddot{x} \\
    \ddot{y}
\end{bmatrix}
+ \begin{bmatrix}
    c_s & 2m_s\gamma\Omega \\
    -2m_s\gamma\Omega & c_s
\end{bmatrix}
\begin{bmatrix}
    \dot{x} \\
    \dot{y}
\end{bmatrix}
+ \begin{bmatrix}
    k_s - m_s\gamma^2\Omega^2 \\
    -\gamma\Omega c_s & k_s - m_s\gamma^2\Omega^2
\end{bmatrix}
\begin{bmatrix}
    x \\
    y
\end{bmatrix} = \vec{h}_s
\]

where the rotational speed of the rotor is \(\Omega\). The relation of \(m_s, c_s, k_s, \gamma\) to the seal geometry, fluid properties and friction factors is well described in \([18]\). Note also that \(c_s, k_s, \gamma\) are nonlinear functions of \(x\) and \(y\), see \([18]\).

1.3 Dynamic Behavior

To analyze the dynamic behavior of the rotor seal system, alongside a time integration simulation, the eigenvalues are also calculated. Therefore we write for the coupled system (rotor + two seals), as in \([14]\):

\[
\begin{bmatrix}
    M & 0 \\
    0 & M
\end{bmatrix}
\begin{bmatrix}
    \ddot{x} \\
    \ddot{y}
\end{bmatrix}
+ \begin{bmatrix}
    C & C \\
    -c & -c
\end{bmatrix}
\begin{bmatrix}
    \dot{x} \\
    \dot{y}
\end{bmatrix}
+ \begin{bmatrix}
    K & k \\
    -k & K
\end{bmatrix}
\begin{bmatrix}
    x \\
    y
\end{bmatrix} = \vec{h}
\]

Where for improved readability the summarized coefficients are:

\[
M = m_r + 2m_s \\
C = 2c_s; \quad c = 4m_s\gamma\Omega \\
K = k_r + 2(k_s - m_s\gamma^2\Omega^2); \quad k = 2\gamma\Omega c_s
\]

It can be seen that the cross coupled terms \(k, c\) lead to tangential forces, which can destabilize the system. These forces transmit energy from the rotor rotation to the bending motion. Assuming a symmetrical rotor system, we substitute \(x\) and \(y\) with the complex coordinates \(z = x + jy\) and \(F_z = h_x + jh_y\), where \(j\) is the imaginary unit. An eigenvalue analysis, for determining the system’s stability and natural frequencies is common; \(F_z = 0\) and \(z = \hat{z}e^{j\omega t}\) are used:

\[
M\lambda^2 + C\lambda + K - j(c\lambda + k) = 0
\]

with the eigenvalues \(\lambda = -\delta + j\omega\). If the decay constant \(\delta\) becomes negative, instability occurs.
1.4 Simulation results

To present the influence of the seals on the rotor system, the simplified Jeffcott model is used with the parameters corresponding to our test rig:

<table>
<thead>
<tr>
<th>Name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotor Shaft</td>
<td>Ø15 × 600 mm</td>
</tr>
<tr>
<td>Mass</td>
<td>5 kg</td>
</tr>
<tr>
<td>1st natural freq.</td>
<td>38.6 Hz</td>
</tr>
<tr>
<td>Seal Diameter</td>
<td>100 mm</td>
</tr>
<tr>
<td>Length</td>
<td>20 mm</td>
</tr>
<tr>
<td>Clearance</td>
<td>0.17 mm</td>
</tr>
<tr>
<td>Pressure</td>
<td>260 kPa</td>
</tr>
<tr>
<td>Dyn. viscosity</td>
<td>0.0405 Pa·s</td>
</tr>
<tr>
<td>Eccentricity</td>
<td>0.2767</td>
</tr>
</tbody>
</table>

The Campbell diagram (see fig. 3) shows the natural frequency, the critical speed, of the ‘dry’ (seal-less) rotor. It can be seen that the first natural frequency is constant, which leads to a critical speed at 2316 rpm. The Campbell diagram (see fig. 4) shows the behavior of the natural frequency influenced by the seals. It follows that the first natural frequency in forward mode $\omega_{1f}$ is lower than half the rotational speed, the backward mode $\omega_{1b}$ is less and can also be seen here. Figure 5 shows the damping ratio $D = \delta / \omega_0$ of the rotor seal system, $\omega_0$ is the undamped natural frequency. The damping ratio $D$ has a zero crossing at 8222 rpm, above which the system becomes unstable. At the onset speed of instability, the rotor starts to vibrate at its first natural frequency, which dominates the displacement response (see waterfall plot of time simulation with unbalance and random noise in fig. 6).

2. EXPERIMENTAL INVESTIGATIONS

This section describes the experimental investigations in analyzing rotor seal systems at the Chair of Applied Mechanics at the Technical University of Munich. First the seals test rig
is presented, then two experimental methods for predicting the onset speed of instability are shown. The first method uses an active magnetic bearing (AMB) for excitation (as a reference method), the second one uses displacement sensor output only data to determine the system’s damping ratio.

2.1 Seals Test Rig

The developed experimental methods are examined on the seals test rig (see fig. 7). The main components are the flexible shaft and a mass disk (1) with two symmetrically arranged plain annular seals (2) in the middle (see details in fig. 8). Eddy current sensors for measuring the displacement (6) and a piezo force platform (7) are arranged in the seals stator housing (8). The fluid is injected between the two seals with a maximum pressure of 100 bar. The rotor runs with over critical speed above the first natural frequency $\omega_1$. An active magnetic bearing (3) is used as an exciter. The rotor shaft is supported by two ball bearings (4) and driven by a servo motor (5). The detailed measurement methodology for seal coefficient determination are described in [17]. The typical test procedure is the stationary rotor run-up (discrete rotational speeds) with and without AMB excitation. Therefore the test rig is controlled and the signals are measured using a DSPACE 1103 system (10 kHz sampling rate).

2.2 AMB Excitation and Transfer Function Measurement

The origin Co-Quad analysis methodology is used for the stability diagnosis for oil-film bearings, see [19] and [20]. The stability analysis method applied to rotor seal systems is clearly described in [21]. Here it is used as a reference method to compare the output only approach.

2.2.1 Analysis Principle

To apply this analysis method, an AMB to excite the rotor seal system with a force $F_z = h_x + j h_y$ in the forward whirl direction (same direction as the rotor rotation $\Omega$) is used. The measured response is the rotor displacement signal $z = x + j y$. Transforming the excitation $F_z$ and the response $z$ into the frequency domain (using $F_z = F_z e^{j \omega t}$ and $z = z e^{j \omega t}$), the transfer function $G(\omega)$ can be calculated as follows:

$$ G(\omega) = \frac{z}{F_z} = \frac{1}{-M \omega^2 + c \omega + K + j(C \omega - k)} $$

(6)

The system becomes unstable ($\dot{z}$ arises), when the denominator of eq. (6) nullifies. This is equivalent to the zero crossing of the damping ratio $D$, shown above. Therefore the transfer function $G$ is separated into real and imaginary part as follows:

$$ Re\{G(\omega)\} = \frac{-M \omega^2 + c \omega + K}{(-M \omega^2 + c \omega + K)^2 - (C \omega - k)^2} $$

$$ Im\{G(\omega)\} = \frac{-C\omega + k}{(-M \omega^2 + c \omega + K)^2 - (C \omega - k)^2} $$

(7)

(8)

In this form, it is possible to determine the zero crossing frequencies $\omega_{re0}$ and $\omega_{im0}$:

$$ 0 = -M\omega_{re0}^2 + c\omega_{re0} + K $$

$$ 0 = -C\omega_{im0} + k $$

(9)

(10)

Solved for $\omega_{re0}$ and $\omega_{im0}$, inserting MUSZYNsKA’s model:

$$ \omega_{re0} = \frac{\gamma}{1 + m_r/2m_s} \Omega $$

$$ \pm \sqrt{\frac{k_r + 2k_s}{m_r + 2m_s} + \frac{4m_s^2\gamma^2}{(m_r + 2m_s)^2 - 2 + m_r/m_s}} \Omega^2 $$

(11)

$$ \omega_{im0} = \frac{\gamma}{\Omega} $$

(12)

The structure of the equations shows that for low rotational speeds, $\omega_{re0}$ is almost equal to the undamped natural frequency $\omega_0$ and exhibits linear behavior at high rotational speeds $\Omega$. $\omega_{im0}$ is linearly proportional to the rotational speed.
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2.2.2 Simulation Results

Figure 9 shows the zero crossing real and imaginary part of the transfer function for the simulation results. Instability occurs at the intersection of both curves (at this point the denominator of $G(\omega)$ nullifies).

2.2.3 Experimental Results

The analysis on the test rig is performed at several constant rotational speeds $\Omega$ (in the save operating range). Then the rotor is excited by the AMB with $F_z$ in forward whirl direction using a sweep (cosine in x, sine in y) about the frequency range of interest (from 10 Hz up to 100 Hz) for calculating the transfer function $G(\omega)$ using fast Fourier transformation (fft). The zero crossing frequencies of the real and imaginary parts of $G(\omega)$, $\omega_{re0}$ and $\omega_{im0}$ must be found (see fig. 10). These zero crossing frequencies show almost linear behavior dependent on the rotational speed $\Omega$, as described above. When both curves cross, the instability occurs. Using a simple linear fit (least squares method) to the points and an extrapolation, the onset speed of instability can be predicted based on measurements in the safe operating range (see fig. 11).

2.3 Damping Ratio Estimation

The damping ratio estimation using output only data is based on the investigations of [11] and [10]. To remove the dominating unbalance response of the rotor system, an interpolation method, presented by [13] for an operational modal analysis case, is applied.

2.3.1 Theoretical Methodology

If the damping ratio becomes zero, the rotor seal system becomes unstable and the amplitudes increase rapidly. The idea of this output only method (without AMB excitation) is to fit the shape of the natural frequency amplitude (see the waterfall plot in fig.12) and to calculate the damping ratio of the system. Therefore a stationary rotor run-up is performed. The displacement signals $x$ and $y$, transformed to $z = x + jy$ are used for the analysis. The advantage of this transformation is its suppression of the backward whirl motion in the positive spectrum to focus on the forward one (the system’s instability is a vibration in the forward whirl motion). To perform curve fitting, the dominating unbalance response at the frequency of the rotational speed $\Omega$, must be removed. For this an interpolation method proposed by Johansson [13] is used. It is possible to fit a PT-2 model (MCK model) curve to the measured amplitude data:

$$|G(\omega)| = \frac{a}{\sqrt{(1 - \frac{\omega}{\omega_1})^2 + (2D \frac{\omega}{\omega_1})^2}}$$

using the first natural frequency $\omega_1$ and the scaling factor $a$. 

Figure 9. Co-Quad analysis of simulation data

Figure 10. Measured real and imaginary part of $G(\omega)$ at 5000 rpm

Figure 11. Co-Quad analysis of measured data
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2.3.2 Experimental Results
Figure 13 shows the fitted model curve, the removed unbalance response and the measured data at 5000 rpm. Figure 14 is a plot of the damping ratio $D$ against the rotational speed $\Omega$ with a linear curve fitted to the data. An extrapolation to the zero crossing leads to the prediction of the onset speed of instability.

3. DISCUSSION AND COMPARISON OF THE APPLIED METHODOLOGIES
In this article, two experimental methods for predicting the onset speed of instability are presented. The predicted rotational speeds for both experimental methods and the bulk flow simulation results are compared in the table:

<table>
<thead>
<tr>
<th>Name</th>
<th>Co-Quad</th>
<th>Output-Only</th>
<th>Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Predicted speed (rpm)</td>
<td>8609</td>
<td>8015</td>
<td>8223</td>
</tr>
</tbody>
</table>

Comparing the waterfall plots of the simulation (just 222 rpm before the instability occurs) and the measurement, fig. 6 and fig. 12, the trend in the increase in the amplitude at the system’s first natural frequency is similar. This implies that at 7000 rpm the test rig is very close to the instability (the peak of the first natural frequency is actually higher than the unbalance response). This behavior shows that the estimated onset speeds of instability are too high compared with the real system. To this extent the presented Output-Only method gives a better prediction of the stability limit than the Co-Quad method.

Additional comments by the authors:
The early stage of the presented methodologies and measurements imply that we cannot make a comment on the uncertainties of the results. Future work will be a sensitivity analysis using different fluid parameters, seal geometries etc. To improve the prediction methodology, the amplitude of...
mass unbalance should be taken into account for the run-up measurement. A higher rotor amplitude leads (due to nonlinear effects in the seal gap, see [4]) to a more stable system and can influence the prediction.

The ability of the MCK model to provide an accurate fit for calculating the damping ratio is well in our simple rotor test rig case, where the first bending mode is the dominating motion. In a real turbomachinery case with multiple resonance peaks the user has to pick out the ‘critical’ natural frequencies (a coupled structure-seal simulation can be used to identify the ‘unstable’ natural frequencies to observe).

To include the output only estimation in a condition monitoring system, only rotor displacement signals have to be observed. The damping ratio and its rotational speed dependency can be estimated during run-up and operation. Observing the available stability margin can be used as a characteristic monitoring value.

4. CONCLUSION

The presented experimental methods for characterizing the stability limit of a rotor seal system are in the stable and safe operating range.

The first method uses an AMB to excite the system at several discrete rotational speeds for measuring the transfer function of rotor displacement to AMB force. An analysis of real and imaginary part according to the rotational speed leads to an estimation of the onset speed of instability.

The second presented method uses Output-Only data (rotor displacement). A PT-2 model fit is used to identify the system’s damping ratio. The rotational speed dependency of the damping ratio leads to an estimation of the stability limit, which can be used as a monitoring value.

The methodologies are examined to a numerical model and to the test rig.

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REFERENCES


