Study on the Influence of Nonlinear Links on the Fuel Control System in Gas Turbine Engine

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Abstract
Fuel system is one of the most important auxiliary systems in a gas turbine engine, whose fuel metering unit and actuator of metering valve have obvious nonlinear links including hysteresis, dead zone, relay and saturation. These nonlinear links have great influence on the fuel control system for gas turbine engine, even on the performance and safety of the whole gas turbine engine. In this paper, the investigations were conducted to analyze the effect of the nonlinear links on the fuel control system. Taking the fuel control system of gas turbine engine as object, a mechanism fuel system model with the nonlinear links was established based on the hydro-mechanical theory using the MATLAB/Simulink platform. The effect of dead zone-relay link of servo amplifier in electro-hydraulic servo actuator was analyzed. The results show that the smaller dead zone in dead zone-relay link causes the oscillation of the metering valve, resulting in the oscillation of the supplying fuel flow rate. The larger dead zone causes the hysteresis and reduces the accuracy of control. By analyzing the reason of oscillation with the describe function method, a fuzzy controller was designed to solve the oscillation problem. The simulation results demonstrate that the designed fuzzy controller can eliminate the negative effects caused by the dead zone-relay link.

Keywords
Fuel system — Dead zone-relay — Oscillation — Fuzzy controller

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INTRODUCTION
Fuel system, controlling the fuel flow rate delivered to the combustor, is one of the most important auxiliary systems in a gas turbine engine. The operating condition of a gas turbine engine is mainly determined by the fuel system. The fuel system regulates the metering valve opening using an electro-hydraulic servo actuator. Commonly, the servo actuator has an own position closed-loop controller, thus the fuel system is a double closed-loop control system in a gas turbine engine. The inner closed-loop can effectively enhance the stability of the fuel control system [1].

![Figure 1 Schematic of the control structure of fuel system](image)

Figure 1 shows the schematic of the control structure of fuel system. Controller 1 is a rotator speed control in the outer loop, which regulates the metering valve position reference according to the rotator speed difference. Controller 2 is a servo actuator position control in the inner loop, which control the metering valve position according the reference from controller 2.

Several kinds of nonlinear links exist in the fuel system, such as the dead zone caused by friction and clearance, the saturation of amplifier and actuator, the dead zone-relay of servo amplifier[2,3]. These nonlinear links have a great influence on the dynamic characteristic of the fuel control system. It subverts the results derived from the linear theoretical analysis. Particularly, the dead zone-relay link would increase the steady state error and response time, even cause the limit cycle oscillation. These negative effects will reduce the accuracy and stability of the fuel control system, resulting in affecting the operating safety and lifetime of the gas turbine engine. Therefore, it is important to study the effect of the nonlinear link on the performance of the fuel system and to know how to weaken or even eliminate the negative effects.

Developing a fuel system model is the foundation of the research of the nonlinear links. In the past few years, most researches simplified the fuel system as a linear system which means that the relationship between valve opening and fuel flow rate is linear or even overlook the fuel system. Liu [4] identified a second order linear model for the metering valve using the autoregressive with exogenous inputs (ARX) method. Madarász[5] et al simplified the electro-mechanical system as a transfer function model. Souami[6] et al established a multi-input multi-output linear identification model using the data-driven method for the fuel system of a jet engine. However, the fuel system has strong nonlinear characteristic. The above linear models are inconsistent with the real fuel system, resulting in the low precision of the linear models. In order to solve the low precision problem, Pan[7] and Wang[8] built the fuel system model based on the hydro-mechanical theory on the Easy5 and AMEsim simulation platform respectively, but
the developed models ignore the servo actuator of the fuel metering valve and the dead zone-relay nonlinear link of servo actuator.

In this paper, a mechanism model, considering the nonlinear links and electro-hydraulic servo actuator, was established based on the hydro-mechanical theory. The effect of the dead zone width of dead zone-relay nonlinear link was simulated and analyzed. In addition, the reason of oscillation due to the dead zone-relay was described and a fuzzy controller was designed to eliminate the oscillation.

1. SYSTEM MODELING

The fuel system of a gas turbine engine consists of fuel metering loop, linear variable differential transformer (LVDT) sensor and an electro-hydraulic servo actuator of metering valve. The schematic of the fuel system is shown in figure 2. The fuel metering loop is composed of filter, pump, metering valve, constant pressure different valve (CPDV) and relief valve. The electro-hydraulic servo actuator accepts the input signal (metering valve opening set value) from the rotator speed controller, and then the servo actuator will cause a movement of the metering valve through a hydraulic cylinder to meet the expected value. Finally, the flow rate of the fuel delivering to the combustor is determined by the metering valve opening.

Figure 2 Schematic of the fuel system in gas turbine engine

In this paper, the component models of the fuel system were developed in the Simscape platform, which is a part of the MATLAB/Simulink library. Simscape is a toolbox specified for modeling the physical systems such as the mechanical, electronic, power and hydraulic systems[9,10]. Furthermore, Simscape also can realize the control algorithm easily. However, the AMESim simulation platform realizes the control algorithm by uniting the Matlab or other platform.

1.1 Fuel pump

The model of the fuel pump is a variable-displacement pump module. The pump delivery equation is shown as following.

\[ q_p = D\omega - k_{\text{leak}}\Delta p_p \]  
(1)

Where \( q_p \) is the pump delivery flow rate, m³/s. \( \omega \) is the angular velocity of the pump, rad/s. \( D \) is the pump instantaneous displacement, m³/rad. \( k_{\text{leak}} \) is the leakage coefficient. \( \Delta p_p \) is the pressure difference between the pump outlet and inlet, Pa.

\[ \Delta p_p = p_2 - p_1 \]  
(2)

Where \( p_2 \) is the outlet pressure, Pa. \( p_1 \) is the inlet pressure, Pa.

The leakage coefficient can be obtained from the following equation.

\[ k_{\text{leak}} = \frac{k_{\text{HP}}}{\nu} \]  
(3)

Where \( \nu \) is the fluid kinematic viscosity, m²/s. \( \rho \) is the fluid density, kg/m³. \( k_{\text{HP}} \) is the Hagen-Poiseuille coefficient.

The Hagen-Poiseuille coefficient can be calculated by the following equation [11].

\[ k_{\text{HP}} = \frac{D\omega_{\text{nom}}(1 - \eta_v)\nu_{\text{nom}}\rho}{\Delta p_{p_{\text{nom}}}} \]  
(4)

Where the subscript \( \text{nom} \) represents the nominal value. \( \eta_v \) is the volumetric efficiency of the pump.

In addition, the leakage flow rate of the fuel pump can be obtained based on the Hagen-Poiseuille theory. The equation is as following.

\[ \Delta p_p = \frac{128\nu PL}{\pi d^4}q_{\text{leak}} = \frac{\nu}{k_{\text{HP}}}q_{\text{leak}} \]  
(5)

Where \( q_{\text{leak}} \) is the leakage flow rate of the pump, m³/s. \( L \) is the length of the leakage path, m. \( d \) is the gap of the leakage path, m.

The input torque of the pump can be calculated by the following equation.

\[ T = \frac{D\Delta p}{\eta_m} \]  
(6)

Where \( T \) is the input torque, N·m. \( \eta_m \) is the mechanical efficiency.

1.2 Metering valve and CPDV

The fuel flow rate across the metering valve can be calculated using the following equation.

\[ q_m = C_m A_m (x_m) \sqrt{2\Delta p_m / \rho} \]  
(7)

Where \( q_m \) is the fuel flow rate across the metering valve, kg/s. \( C_m \) is the flow coefficient. \( A_m \) is the orifice area, mm². \( x_m \) is the metering valve opening, mm. \( \Delta p_m \) is the pressure difference of the metering valve outlet and inlet, Pa.

Figure 3 Schematic of CPDV

The fuel flow rate of the metering valve is related to the orifice area and the pressure difference according to the equation (7). A CPDV is introduced to maintain the pressure difference by regulating the opening of the CPDV [8]. Therefore, the fuel flow rate is only related to the orifice area and the relationship between the fuel flow rate and orifice area is linear.

The schematic of the CPDV is shown in figure 3. The
force balance equation is as following.

\[(P_{1, pd} - P_{2, pd})s_{pd} = F_{s, pd} + k_{s, pd}x_{pd}\]

\[F_{1, pd} - F_{2, pd} = F_{s, pd} + k_{s, pd}x_{pd}\]  \(8\)

Where \(s_{pd}\) is the force area of CPDV, \(m^2\); \(F_{s, pd}\) is the preload of the spring, \(N\); \(k_{s, pd}\) is the spring stiffness, \(N/mm\); \(x_{pd}\) is the opening of CPDV, \(mm\); \(P_{1, pd}\) is the inlet pressure of CPDV, \(Pa\); \(P_{2, pd}\) is the inlet pressure of CPDV, \(Pa\); \(F_{1, pd}\), \(P_{1, pd}\), \(s_{pd}\) is the inlet fluid force, \(N\); \(F_{2, pd}\), \(P_{2, pd}\), \(s_{pd}\) is the outlet fluid force, \(N\).

When the opening of the metering is changed, the pressure difference of the metering will also be changed. The force balance in CPDV is broken and the unbalance force drives the CPDV spool move to a new balance place. The CPDV model is established in simscape according to the operating theory, shown in figure 4. The pressure difference module is used to calculation the force resulting from the pressure difference. The force sensor module is used to calculate the force of the spring. The CPDV spool model is built with mass, spring, and damper modules. The inputs are the inlet pressure and outlet pressure of the metering valve, and output is the opening of the CPDV.

1.3 Electro-hydraulic servo actuator

The electro-hydraulic servo actuator is used to control the position of the metering valve. It is composed of hydraulic cylinder, electro-hydraulic servo valve, servo amplifier, LVDT and the auxiliary parts [12]. The electro-hydraulic servo actuator model in simscape is shown in figure 5, including electro torque motor model, servo valve model, hydraulic cylinder model, and metering valve spool model. A double-acting hydraulic cylinder model is built to driven the metering valve spool which is modeled with mass, spring, and damper modules. The hydraulic cylinder is controlled by a servo valve model.

There are several nonlinear links in the actual physical system.
device. For example, the hydraulic cylinder has dead zone and saturation nonlinear link, the servo amplifier has dead zone-relay and saturation nonlinear link. All these nonlinear links were considered in the model. For the dead zone-relay nonlinear link, the relationship between output and input are shown in figure 6.

As the figure 6 shown, the dead zone-relay link has three different output values which include \(-M\), 0 and +M. When the input falls into the dead zone (2d), the output is 0. Only if the input is more than the threshold, the output is near-maximum. Therefore, the equation of the dead zone-relay is as following [13].

\[
u(x) = \begin{cases} 
M, & x > d \\
0, & -d < x < d \\
-M, & x < -d 
\end{cases}
\]  \hspace{1cm} (9)

2. ANALYSIS OF DEAD ZONE-RELAY EFFECT

The mechanism model of the fuel system considering the nonlinear links was established in simscape (see in figure 7) according to the layout presented in figure 2.

2.1 Effect of the dead zone width

In order to analyze the effect of the dead zone-relay nonlinear link, the dynamics of the fuel system under different dead zone width were investigated when the metering valve opening reference steps from 19mm to 15mm.

The dynamics of the metering valve opening, fuel flow rate and the metering valve pressure difference are shown in figure 8-10. It can be seen that the dead zone width of the dead zone-relay link has a significant effect on the fuel control system. The fuel system will appear oscillation when the dead zone width is too small. The system will be stable with the increase of dead zone width, but causing the increase of the steady state error.

As the results shown in figure 8, the metering valve opening will appear oscillation when the dead zone width is 0.0005, and the fluctuation value of metering valve opening is 1.1mm. When the dead zone width is increased to 0.001, the dynamic response of the metering valve opening is stable and the steady state error is only 0.53%. The steady state error increases to 2.6% with the dead zone width increased to 0.002.

Figure 7 Mechanism model of the fuel system in simscape

Figure 8 Dynamic response of the metering valve opening

The dynamics of the metering valve opening, fuel flow rate and the metering valve pressure difference are shown in figure 8-10. It can be seen that the dead zone width of the dead zone-relay link has a significant effect on the fuel control system. The fuel system will appear oscillation

Figure 9 Dynamic response of the fuel flow rate

Figure 9 demonstrates the dynamic response of the fuel flow rate. The trend of the fuel flow rate is similar to the metering valve opening with different dead zone width. The fluctuation value of metering valve opening is about
0.193kg/s when the dead zone width is 0.0005.

**Figure 10** Dynamic response of the metering valve pressure difference

The results of figure 10 report that the CPDV can effectively maintain the pressure difference of the metering valve at 0.25MPa. Thus, the CPDV model in the simscape can work successfully.

It is noted that the fuel system may be oscillating due to the dead zone-relay nonlinear link. The cyclic variation of the fuel flow rate leads to the gas turbine engine operating under the cyclic load condition, which affects the safety and shortens the lifetime of the gas turbine engine. Thus, it is necessary to solve this problem by analyzing the reason of oscillation.

2.2 Reason of oscillation

In order to analyze the effect of the dead zone-relay nonlinear link using the describing function method, the fuel control loop (position closed-loop control in figure 2) is equivalent to a nonlinear system shown in figure 11.

**Figure 11** Equivalent schematic of position closed-loop control

The describing function of the dead zone-relay nonlinear link is as following [14-16].

\[
N(X) = \frac{4M}{\pi X} \sqrt{1 - \left(\frac{d}{X}\right)^2}, \quad X \geq d
\]

(10)

Where \(N(X)\) is the describing function of the dead zone-relay link. \(X\) is the sinusoidal input amplitude.

The locus of inversion of the describing function \(N(X)\) and the Nyquist plot of \(G(s)\) are shown in figure 12.

**Figure 12** Nyquist plot of \(G(jw)\) and locus of \(-1/N(X)\)

If the Nyquist plot of \(G(jw)\) (curve 1) does not cross the \(-1/N(X)\) locus, the system is stable according to the Nyquist stability criterion. Conversely, if the Nyquist plot of \(G(jw)\) (curve 2) crosses the \(-1/N(X)\) locus, the system will appear oscillation due to the limit cycle. Therefore, when the dead zone width \(d\) increases, the point A in figure 10 will move to the left until the Nyquist plot of \(G(jw)\) does not cross the \(-1/N(X)\) locus, resulting to eliminate the oscillation. Therefore, a larger dead zone width causes a more stable system but leads to a larger steady state error.

3. FUZZY COMPENSATION METHOD

The dead zone is fixed and cannot be adjusted directly for a working physical device. Many compensation strategies have been proposed to eliminate the negative influence of the dead zone-relay. On the whole, the compensation methods include hardware compensation and software compensation. Hardware compensation needs additional hardware device, which is easily interfered by other devices causing the low compensation precision. However software compensation uses the compensation algorithms instead of the hardware compensation device, including low cost, high stability and strong generalization capability [17,18]. In this paper, a one dimensional fuzzy compensation controller was design for the software compensation [19-21]. The schematic of the fuzzy compensation is shown in figure 13.

**Figure 13** Schematic of fuzzy compensation controller

According to the layout shown in figure 13, the input of the designed fuzzy controller is error between the metering valve position from LVDT and the position reference from the rotator speed controller. The fuzzy controller has five linguistic variables: Negative Large (NL), Negative Small (NS), Zero (ZO), Positive Small (PS) and Positive Large (PL). The membership functions for the input and output of the fuzzy compensation controller are shown in figure 14.. The domains of the fuzzy input and output are both [-4, 4].
The fuzzy rules of the fuzzy controller are as follows:

IF Input is NL, then Output is NL,
IF Input is NS, then Output is NS,
IF Input is ZO, then Output is ZO,
IF Input is PS, then Output is PS,
IF Input is PL, then Output is PL.

The output curve of the fuzzy controller is shown in figure 15.

The designed fuzzy controller was added into the position closed-loop control. In addition, the dead zone width of the dead zone-relay link was set to 0.0005 V. The dynamic response of the fuel system was analyzed when the metering valve opening reference steps from 19mm to 15mm after fuzzy compensation.

The dynamic responses of the metering valve opening, fuel flow rate and the metering valve pressure difference after fuzzy compensation are shown in figure 16-18, respectively.

The figures demonstrate that the fuel system does not appear oscillation with the 0.0005 dead zone width when the metering valve opening reference has a step decrease. Therefore, the designed fuzzy controller can effectively eliminate the oscillation caused by the dead zone-relay nonlinear link.

4. CONCLUSIONS

A mechanism fuel system model considering several nonlinear links was established in the Simscape platform. Particularly, the effect of the dead zone-relay nonlinear link under different dead zone width was analyzed. In addition, the compensation method of the nonlinear link was also investigated to eliminate the negative influence. The conclusions are as following.

(1) A mechanism model of the fuel control system considering the nonlinear links was developed based on the hydro-mechanical theory in the Simscape platform.

(2) The effect of the dead zone-relay nonlinear link with different dead zone width on the fuel control system dynamic performance was analyzed. The results demonstrate that the dead zone width has a great influence for the fuel control system. When the dead zone width is too small, the fuel system will be oscillating. The fuel control system will be stable with the increase of dead zone width, but causing the increase of the steady state error.
(3) A one dimensional fuzzy compensation controller is designed to eliminate the oscillation. The simulation results show that the designed fuzzy controller can effectively eliminate the oscillation when the dead zone width is too small.

NOMENCLATURE

\begin{align*}
A & \quad \text{orifice area [mm}^2]\text{]}
\hline
C & \quad \text{flow coefficient.}
CPDV & \quad \text{constant pressure difference valve}
d & \quad \text{gap of the leakage path [m], dead zone width}
D & \quad \text{pump instantaneous displacement [m}^3/\text{rad]}
F & \quad \text{force [N]}
G & \quad \text{transfer function}
k & \quad \text{coefficient, spring stiffness [N/mm]}
L & \quad \text{length of the leakage path [m]}
LVDT & \quad \text{linear variable differential transformer}
M & \quad \text{output of dead zone-relay}
N & \quad \text{describing function}
p & \quad \text{pressure [Pa]}
q & \quad \text{flow rate [m}^3/\text{s]}
s & \quad \text{force area [m}^2]\text{]}
T & \quad \text{torque [N-m]}
U & \quad \text{output of the dead zone-relay link}
x & \quad \text{valve opening [mm], input of dead zone-relay}
\chi & \quad \text{the sinusoidal input amplitude}
\omega & \quad \text{angular velocity, [rad/s]}
\nu & \quad \text{fluid kinematic viscosity [m}^2/\text{s]}
\rho & \quad \text{fluid density [kg/m}^3]\text{]}
\eta_m & \quad \text{mechanical efficiency}
\eta_v & \quad \text{volumetric efficiency}
\Delta & \quad \text{difference}
\hline
\end{align*}

\begin{align*}
1 & \quad \text{inlet}
2 & \quad \text{outlet}
HP & \quad \text{Hagen-Poiseuille}
\text{leak} & \quad \text{leakage parameter}
m & \quad \text{metering valve}
\text{nom} & \quad \text{nominal value}
p & \quad \text{pump}
pd & \quad \text{CPDV}
\end{align*}

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REFERENCES


